Detailed analysis of the solar orbital

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References

1. **E.M.Standish** - *Keplerian Elements for Approximate Positions of the Major Planets*, JPL Caltech
2. **Farshad Barman** - *Astronomy Projects for Calculus and Differential Equations* 2012
3. **H. Karttunen** and al. - *Fundamental Astronomy* 4 Ed. Springer 2007
4. http://objectual-philosophy.com/application/PlanetaryOrbitals.pdf
5. http://objectual-philosophy.com/application/The Solar orbital and solar activity.pdf
6. http://objectual-philosophy.com/book/counter1.php (Introduction into Objectual Philosophy.pdf)
7. **Roy Martin** - *How do the planets affect the Sun ?* March 17, 2013
8. **Arvind Bhatnagar, William Livingston** - *Fundamental of Solar Astronomy* World Scientific Publishing Co 2005.

1 - Mathematical modelling

1.1 - Introduction

To begin with, let us remind the reader what an *orbital* is, a notion described in detail in [4]: An orbital of an astronomical body (AB) belonging to a planetary system (PS) has the following invariant attributes in its composition:

– The spatial area in which the orbital motion is inscribed (defined by the semi-major axis of the orbit, eccentricity and inclination of the orbital plane), attributes defined in relation to the centre of mass (CM) of the PS and in relation to the equatorial plane of the Sun;

– Orbital period or its inverse - *orbital frequency*.

Unlike the previous research [5], in which the motion of the planets determining the solar motion was modelled purely theoretically, without a direct connection with the actual motion of the planets, in this research paper we will use an approximation method for the position of the planets indicated in [1], which describes the calculation method for the positions of the planets using ephemerides valid in certain time intervals, from which we chose the table with the longest interval (3000BC-3000AC). To solve Kepler’s equation, however, we used a method different from the one in [1], a method indicated in [2] in which the solutions of the Kepler’s equation are approximated with Bessel functions of first or second order. Although in [1] it is expressly specified that in the case of giant planets the corrections indicated in table 1.5.1.B will be used to calculate the mean anomaly M, we will initially neglect these corrections for reasons that we will discuss towards the end of this research.

1.2 - Ellipse as a projection of a circle

Let a circle with *a* radius be in an XY plane, where the X axis is horizontal and the Y axis is vertical, the origin being in the centre of the circle. Let XY' be a second plane inclined at an angle φ from the XY plane around the X axis. If we project the circle from the XY plane to the XY' plane, we obtain an ellipse with the *a* major axis collinear to the X axis and the minor axis:

$$b = a \cdot \cos(\varphi) \qquad (1.2.1)$$

collinear to the Y' axis. In an ellipse there is the equation:

$$\varepsilon^2 = a^2 - b^2 \qquad (1.2.2)$$

where ε is the *linear eccentricity* or *focal length*. The value:

$$e = \frac{\varepsilon}{a} \qquad (1.2.3)$$

is called *numerical eccentricity*. If we replace equations 1.2.1 and 1.2.3 in 1.2.2 we will obtain:

$$e^2 = 1 - \cos^2(\varphi) \qquad (1.2.4)$$

i.e.:

$$e = \sin(\varphi) \qquad (1.2.5)$$

or:

$$\varphi = \arcsin(e) \qquad (1.2.6)$$

From equation 1.2.6 we can determine φ for the trajectories of current planets, with *e* (eccentricity of the orbit) being known from the tables with the orbit data, and from equation 1.2.1 we can determine *b*, with *a* (semi-major axis of the orbit) being known. The result of the equations 1.2.2 and 1.2.3 is:

$$b_1 = a\sqrt{1-e^2} \qquad (1.2.7)$$

and the result of the equations 1.2.1 and 1.2.6 is:

$$b_2 = a\cos(\arcsin(e)) \qquad (1.2.8)$$

Take, for example, the case of Earth, where $a = 1UA$, $e = 0.01671022$. With these values it follows that $b_1 = b_2 = 0.9998603745$ *AU*, a proof that equations 1.2.7 and 1.2.8 are equivalent.

Comment1.2.1: The whole paragraph 1.2 was intended to justify the replacement of the expression $\sqrt{1-e^2}$ with $\cos(\arcsin(e))$ in the equations from the next paragraph.

1.3 - Motion on an elliptical trajectory

If the radius vector of the *a* radius circle performs a uniform rotational motion with the *T* period, the angle between the radius vector and the X axis is called (in the case of astronomers [3]) an *eccentric anomaly E*. On the elliptical trajectory resulting from the projection of the *a* radius circle, the radius vector of the planet $r(x, y)$ originating in a focus will perform a rotation with uneven speed, but with the same period *T* (*t* being the time from the moment of passing through the perihelion); in this case the *v* angle of the radius vector of the planet with respect to the X axis is called *true anomaly*, and the value:

$$M = \frac{2\pi}{T} \cdot t \qquad (1.3.1)$$

is called *mean anomaly*. There is a relationship between these values called *Kepler's equation*:

$$M = E - e \cdot \sin(E) \qquad (1.3.2)$$

Comment 1.3.1: The term $e \cdot \sin(E)$ is dimensionless (numerical), but the dimensional analysis of equation 1.3.2 tells us that this term must have the dimension of an angle, so in this case *e* will be assigned either a dimension in radians (unchanged numerical value) or in degrees by multiplying by $180/\pi$ (see [1]).

Equation 1.3.2 is a transcendental equation, the solutions of which (according to [2]) were found by Friedrich Bessel in the form of a power series:

$$E = M + 2 \cdot \sum_{n=1}^{\infty} \frac{J_n(n \cdot e)}{n} \sin(M) \qquad (1.3.3)$$

where $J_n(x)$ is the Bessel function of order *n*:

$$J_n(x) = \sum_{i=0}^{\infty} \frac{(-1)^i \cdot x^{2i+n}}{i! \cdot (i+n)! \cdot 2^{2i+n}} \tag{1.3.4}$$

For small eccentricities ($e \le 0.21$, the case of the planets in our planetary system) only the first term of the series (e) is needed to keep the orbital position error less than 4.1%. For better accuracy the second term is $e^2/2$. Finally, the equations that give the positions of a planet as a function of time with respect to the X axis (*perihelion-aphelion axis*) in case of ignoring the Kepler equation are:

$$\begin{aligned} x(t) &\cong a \cdot \cos(M) - a \cdot e = a \cdot [\cos(M) - e] \\ y(t) &\cong b \cdot \sin(M) = a \cdot \sin(M) \cdot \cos(\arcsin(e)) = a \cdot \sin(M) \cdot \sqrt{1-e^2} \end{aligned} \tag{1.3.5}$$

or taking into account the Kepler equation for $e \le 0.21$:

$$\begin{aligned} x(t) &\cong a \cdot \cos[M + e \cdot \sin(M)] - a \cdot e = a \cdot [\cos[M + e \cdot \sin(M)] - e] \\ y(t) &\cong a \cdot \sin[M + e \cdot \sin(M)] \cdot \cos(\arcsin(e)) \end{aligned} \tag{1.3.6}$$

and finally, for a better approximation:

$$\begin{aligned} x(t) &\cong a \cdot [\cos[M + e \cdot \sin(M) + \frac{e^2}{2} \cdot \sin(2M)] - e] \\ y(t) &\cong a \cdot \sin[M + e \cdot \sin(M) + \frac{e^2}{2} \cdot \sin(2M)] \cdot \cos(\arcsin(e)) \end{aligned} \tag{1.3.7}$$

Please note! In equations 1.3.6, 1.3.7 the correction indicated in comment 1.3.1. applies to e or $e^2/2$. Note that these equations are valid for the motion of a planet on a heliocentric elliptical trajectory under special conditions, i.e. the apse line (perihelion-aphelion axis) is collinear to the X axis, and the start time is the moment of passing through the perihelion.

The fact that the actual trajectories of the planets in our solar system are not coplanar has not been taken into account either, however, the differences in inclination with respect to the solar equatorial plane of the orbits of the giant planets[1] are less than 0.1 radians, therefore they can be neglected.

1.4 - Motion of PS elements with respect to CM of PS (in barycentric coordinates)

Let a PS be represented in fig. 1.4.1 consisting of *n* small *astronomical bodies* (ABs) (planets), with masses $m_1, m_2, ...m_n$ orbiting a large AB (Sun) with mass m_S. In relation to an external 2D reference system with O origin and XY axes, the PS elements have spatial position vectors $\overline{r}_1, \overline{r}_2, \ldots \overline{r}_n$ and $\overline{r}_S$.

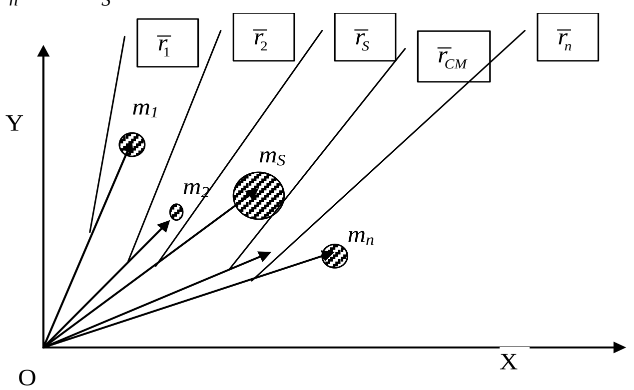

Fig. 1.4.1

[1] For long-term analyses of the parameters of the solar orbital, only the motions of the four giant planets are important, the rest of the planets having (with a few exceptions) only minor contributions.

Under these conditions, the centre of mass CM of the PS has the position vector $\overline{r}_{CM}$ given by the equation:

$$\overline{r}_{CM} = \frac{m_1\overline{r}_1 + m_2\overline{r}_2 + \ldots + m_n\overline{r}_n + m_S\overline{r}_S}{m_1 + m_2 + \ldots + m_n + m_S} \qquad (1.4.1)$$

If we move the O origin of the reference system to CM, this is equivalent to writing in the equation 1.4.1 $\overline{r}_{CM} = 0$. By noting $m_1 + m_2 + \ldots + m_n + m_S = m_T$, where m_T is the total mass of the PS, the equation 1.4.1 becomes:

$$\frac{m_S}{m_T}\overline{r}_S = -\frac{m_1}{m_T}\overline{r}_1 - \frac{m_2}{m_T}\overline{r}_2 - \ldots - \frac{m_n}{m_T}\overline{r}_n \qquad (1.4.2)$$

hence the position vector of the Sun with respect to CM:

$$\overline{r}_S = -\frac{m_1}{m_S}\overline{r}_1 - \frac{m_2}{m_S}\overline{r}_2 - \ldots - \frac{m_n}{m_S}\overline{r}_n \qquad (1.4.3)$$

By noting $\frac{m_S}{m_i} = q_i \;\; i \in [1, n]$, equation 1.4.3 becomes:

$$\overline{r}_S = -\sum_{i=1}^{n}\frac{\overline{r}_i}{q_i} \qquad (1.4.4)$$

1.5 - Actual motion of a planet in heliocentric coordinates

In the heliocentric coordinate system the reference plane is the equatorial plane of the Sun in which the reference axis X_{ecl} is the axis of the equinoxes (see fig. 1.5.1). The values involved in defining planetary orbits in this reference system are:

- Ω the longitude of the ascending node AN;
- ω the argument of perihelion;
- $\varpi = \omega + \Omega$ the longitude of the perihelion;
- M is the mean anomaly;
- v it is the true anomaly;
- r is the radius vector of the planet;
- $L = M + \omega + \Omega$ is the mean longitude

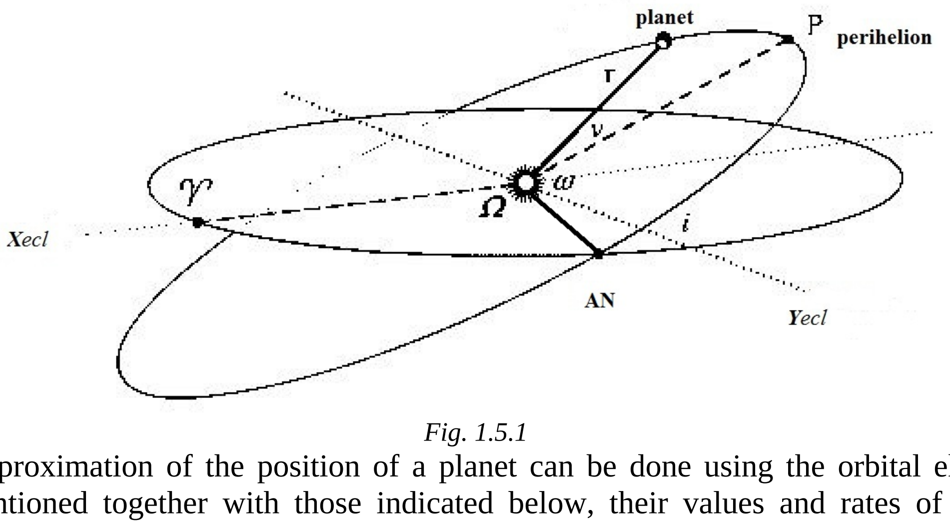

Fig. 1.5.1

The approximation of the position of a planet can be done using the orbital elements already mentioned together with those indicated below, their values and rates of secular variation being given in tables 1.5.1.A and 1.5.1.B, values valid for the time interval 3000BC - 3000AD.

The orbital elements together with their secular variations are:

- $a_0, \dot{a}$ semi-major axis of the orbit [AU, AU/century];

- $e_0, \dot{e}$ eccentricity [, /century];
- $i_0, \dot{i}$ orbit inclination [degrees, degrees/century];
- $L_0, \dot{L}$ mean longitude [degrees, degrees/century];
- $\varpi_0, \dot{\varpi}$ perihelion longitude [degrees, degrees/century];
- $\Omega_0, \dot{\Omega}$ ascending node longitude [degrees, degrees/century];

For the calculation of the orbital elements we calculate $a = a_0 + \dot{a} \cdot t$, $e = e_0 + \dot{e} \cdot t$ etc. where *t* is the number of centuries past since the day $t_{efe} = J2000.0$.

With the data in tables 1.5.1.A and 1.5.1.B we then calculate *M* with the equation:

$$M = L - \varpi + b \cdot t^2 + c \cdot \cos(f \cdot t) + s \cdot \sin(f \cdot t) \quad (1.5.1)$$

Now we can determine the position vector $r(t) = x(t) + y(t)$ of the planet in the plane of its orbit (the X axis being the apse line) with the equations 1.3.6 or 1.3.7. Next, by rotating the coordinate axes we will obtain the ecliptic coordinates of the planets (see fig. 1.5.1) with the equations:

$$xe(t) = (\cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i) \cdot x(t) + (-\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i) \cdot y(t) \quad (1.5.2)$$

$$ye(t) = (\cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i) \cdot x(t) + (-\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i) \cdot y(t) \quad (1.5.3)$$

$$ze(t) = (\sin\omega\sin i) \cdot x(t) + (\cos\omega\sin i) \cdot y(t) \quad (1.5.4)$$

Given the small inclination of the orbital planes, especially for the giant planets, we will consider $i = 0$, so only the equations 1.5.2 and 1.5.3 will remain (in the reduced version):

$$xe(t) = (\cos\omega\cos\Omega - \sin\omega\sin\Omega) \cdot x(t) + (-\sin\omega\cos\Omega - \cos\omega\sin\Omega) \cdot y(t) \quad (1.5.5)$$

$$ye(t) = (\cos\omega\sin\Omega + \sin\omega\cos\Omega) \cdot x(t) + (-\sin\omega\sin\Omega + \cos\omega\cos\Omega) \cdot y(t) \quad (1.5.6)$$

where *xe* and *ye* are the ecliptic coordinates of the planets along the axes X_{ecl} şi Y_{ecl} in Fig. 1.5.1.

Comment 1.5.1: It should be noted that in equations 1.5.5 and 1.5.6 the planetary position vectors are calculated in heliocentric ecliptic coordinates, while the position vector of the Sun given by equation 1.4.4 is calculated in the coordinates of the centre of mass of PS (barycentric). Given that the distance from the sun to the CM can reach up to $1.58 \cdot 10^6$ km, this means a maximum error of 2.73% for Mercury's position vector, but a much smaller error for the positions of the giant planets that have the largest contribution to the solar position relative to CM.

Table 1.5.1.A

Planet	$a_0, \dot{a}$ *au,au/cty*	$e_0, \dot{e}$ *, /cty*	$i_0, \dot{i}$ *deg.,deg./cty*	$L_0, \dot{L}$ *deg.,deg./cty*	$\varpi_0, \dot{\varpi}$ *deg.,deg./cty*	$\Omega_0, \dot{\Omega}$ *deg.,deg./cty*
Me	0.38709843 0.00000000	0.20563661 0.00002123	7.00559432 -0.00590158	252.25166724 149472.67486623	77.45771895 0.15940013	48.33961819 -0.12214182
Ve	0.72332102 -0.00000026	0.00676399 -0.00005107	3.39777545 0.00043494	181.97970850 58517.81560260	131.76755713 0.05679648	76.67261496 -0.27274174
Ea	1.00000018 -0.00000003	0.01673163 -0.00003661	-0.00054346 -0.01337178	100.46691572 35999.37306329	102.93005885 0.31795260	-5.11260389 -0.24123856
Ma	1.52371243 0.00000097	0.09336511 0.00009149	1.85181869 -0.00724757	-4.56813164 19140.29934243	-23.91744784 0.45223625	49.71320984 -0.26852431
Ju	5.20248019 -0.00002864	0.04853590 0.00018026	1.29861416 -0.00322699	34.33479152 3034.90371757	14.27495244 0.18199196	100.29282654 0.13024619
Sa	9.54149883 -0.00003065	0.05550825 -0.00032044	2.49424102 0.00451969	50.07571329 1222.11494724	92.86136063 0.54179478	113.63998702 -0.25015002
Ur	19.18797948 -0.00020455	0.04685740 -0.00001550	0.77298127 -0.00180155	314.20276625 428.49512595	172.43404441 0.09266985	73.96250215 0.05739699
Ne	30.06952752 0.00006447	0.00895439 0.00000818	1.77005520 0.00022400	304.22289287 218.46515314	46.68158724 0.01009938	131.78635853 -0.00606302
Pl	39.48686035 0.00449751	0.24885238 0.00006016	17.14104260 0.00000501	238.96535011 145.18042903	224.09702598 -0.00968827	110.30167986 -0.00809981

Additional terms to be added to the calculation of *M* for giant planets, in the same time interval 3000BC - 3000AD.

Table 1.5.1.B

Planet	b	c	s	f
Ju	-0.00012452	0.06064060	-0.35635438	38.35125
Sa	0.00025899	-0.13434469	0.87320147	38.35125
Ur	0.00058331	-0.97731848	0.17689245	7.67025
Ne	-0.00041348	0.68346318	-0.10162547	7.67025

2 - Main distributions of the solar orbital

2.1 - Primary spatiotemporal distribution[2] of the position of the Sun

As mentioned in the introduction of this research, for the beginning we will disregard the corrections introduced by table 1.5.1.B, this fact meaning that the mean anomaly M of a planet will be calculated with the equation:

$$M = L - \varpi \tag{2.1.1}$$

valid for both terrestrial and giant planets. We will discuss the reasons for this disregard towards the end of this research.

The primary temporal distribution of the position of the Sun in its motion around the centre of mass of the planetary system (CM) is given by the equations:

$$xes(t) = -\sum_{i=0}^{7} \frac{xe(i,t)}{q_i} \cdot AU \;;\; yes(t) = -\sum_{i=0}^{7} \frac{ye(i,t)}{q_i} \cdot AU \quad [km] \tag{2.1.2}$$

where *xes* is the solar coordinate along the ecliptic axis X_{ecl} (axis of equinoxes, see fig. 1.5.1), *yes* is the solar coordinate along the axis Y_{ecl}, *xe(i,t)* and *ye(i,t)* are the ecliptic coordinates of the planet *i* $(i \in [0,7])$ as a function of *t* (given by equations 1.5.5 and 1.5.6), and q_i is the ratio of the mass of the Sun and the mass of the planet *i*. $AU = 1.4959787 \cdot 10^8$ km is the astronomical unit of distance. Time *t* is expressed in centuries relative to the reference year J2000, the time increment being Nd=8.359990576 days ([3]). The graphical representation of equations 2.1.2 in linear Cartesian coordinates is given in fig. 2.1.1.

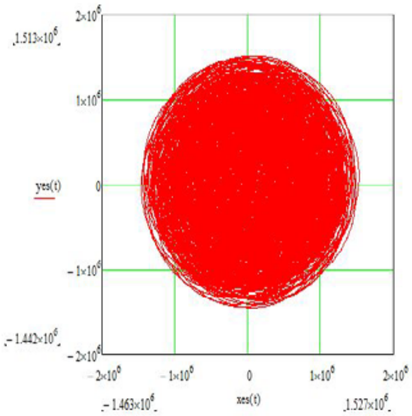

Fig. 2.1.1 Solar orbital in linear Cartesian coordinates

From Fig. 2.1.1 it can be seen that the space domain of the solar orbital is occupied almost uniformly, but the observation is not correct - in the immediate vicinity of the CM (with coordinates 0.0) the space domain[4] is void. This aspect is visible in the logarithmic Cartesian representation in fig. 2.1.2.

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[2] The names of the distributions are those introduced in chap. 2 of the research [6].

[3] The time increment *Nd* days resulted from the division of the time interval of 60 centuries in which the ephemerides table is valid (3000BC-3000AC) with the maximum number of samples allowed by the software used for modeling (Mathcad 14) for graphs (2^{18}).

[4] In the vicinity of this domain, the radial acceleration of the Sun reaches very high values.

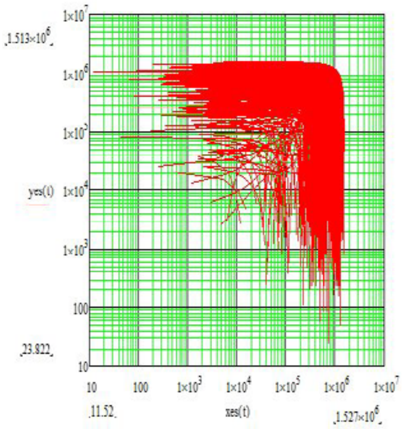

Fig. 2.1.2 Solar orbital in logarithmic Cartesian coordinates

From fig. 2.1.2 we see that the space area with a radius less than approx. $2 \cdot 10^3$ km relative to CM is unoccupied, i.e. the proximity of the Sun to this area is extremely rare.

2.2 – First-order derived distribution of the solar position

The components of the derived time distribution of the solar position (orbital velocity) are:

$$vxes(t) = \frac{xes(t) - xes(t - \Delta t)}{Nd} \,;\, vyes(t) = \frac{yes(t) - yes(t - \Delta t)}{Nd} \quad \left[{km}/{day} \right] \qquad (2.2.1)$$

where *xes(t)* and *yes(t)* are given by equations 2.1.2, and $\Delta t = 2.28884326 \cdot 10^{-4}$ is the time increment in centuries corresponding to the *Nd* days. The graphical representation of equations 2.2.1 is given in fig. 2.2.1.

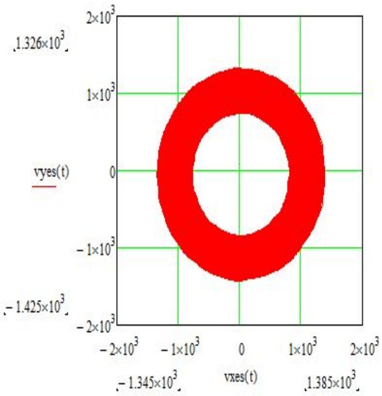

Fig. 2.2.1 Time distribution of solar orbital velocity around the CM

Comment 2.2.1: The coloured area in Fig. 2.2.1 represents the range of solar orbital velocity values, a range between two limits: outer and inner. The outer limit, with the values marked on the figure, consists of Jupiter's contribution to the orbital velocity of the Sun (shown in Fig. 3.1.2) plus the sum of the contributions of the other planets (especially the giant ones) to the orbital velocity of the Sun presented in par. 3. The inner limit also consists of Jupiter's contribution to the orbital velocity of the Sun minus the sum of the contributions of the other planets. This foreshadows the major role of the planet Jupiter in the processes that take place in our solar system.

Time distribution of the solar orbital velocity module $ves(t) = \sqrt{vxes(t)^2 + vyes(t)^2}$ is shown in Fig. 2.2.2, the X axis being graduated in centuries (the value 0 corresponds to the year J2000).

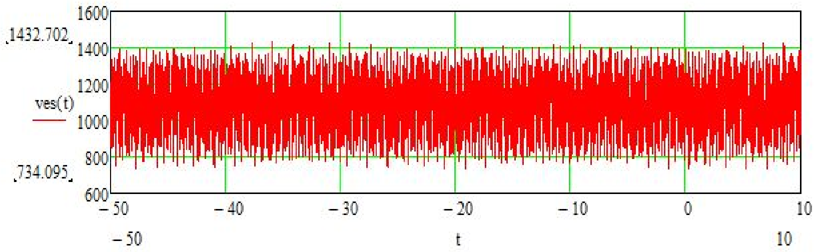

Fig. 2.2.2 Time distribution of the solar orbital velocity module.

2.3 – Second-order derived distribution of the solar position

The components of the derived time distribution of the solar velocity (solar orbital acceleration) are:

$$axes(t) = \frac{vxes(t) - vxes(t - \Delta t)}{Nd}\ ;\ ayes(t) = \frac{vyes(t) - vyes(t - \Delta t)}{Nd} \quad \left[{}^{km}\!/_{day^2} \right] \qquad (2.3.1)$$

and the graphical representation:

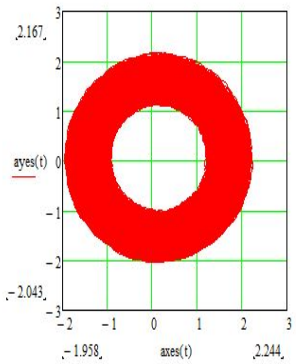

Fig. 2.3.1 Distribution of solar orbital acceleration around CM

Comment 2.3.1: In the case of solar orbital acceleration, there is also a domain in which the values of this solar parameter fall, a domain between the two limits: outer and inner. As in the case of the solar orbital velocity, the outer limit with the values marked on the figure is given by Jupiter's contribution to the orbital acceleration of the Sun (shown in Fig. 3.1.4) plus the sum of the other planetary contributions. The inner limit is also given by the Jovian contribution to the orbital acceleration of the Sun minus the sum of the other planetary contributions to this parameter.

Temporal distribution of the solar orbital acceleration module $aes(t) = \sqrt{axes(t)^2 + ayes(t)^2}$ is shown in Fig. 2.3.2:

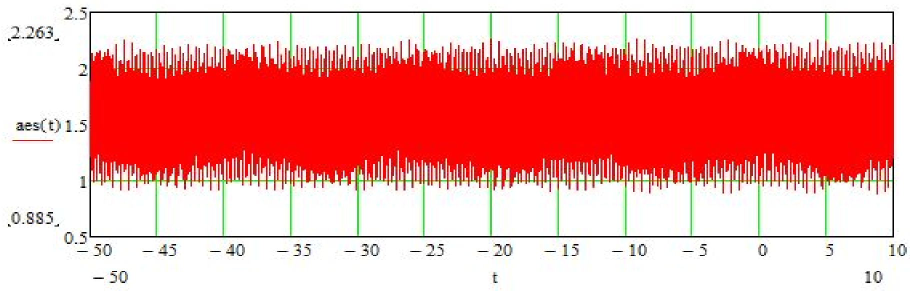

Fig. 2.3.2 Time distribution of the solar orbital acceleration module

2.4 - Distributions of solar distance from CM

The distance of the Sun from the CM is given by the module of the solar position vector:

$$res(t) = \sqrt{xes(t)^2 + yes(t)^2} \qquad (2.4.1)$$

(see Fig. 2.4.1 a and b) and the derived distributions of this distance are:

$$vres(t) = \frac{res(t) - res(t - \Delta t)}{Nd} \qquad (2.4.2)$$

for the solar radial velocity (see Fig. 2.4.2) and:

$$ares(t) = \frac{vres(t) - vres(t - \Delta t)}{Nd} \qquad (2.4.3)$$

for solar radial acceleration (see Fig. 2.4.3).

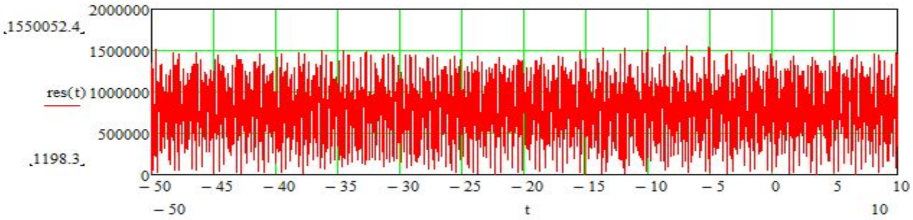

Fig. 2.4.1.a Linear solar position vector module

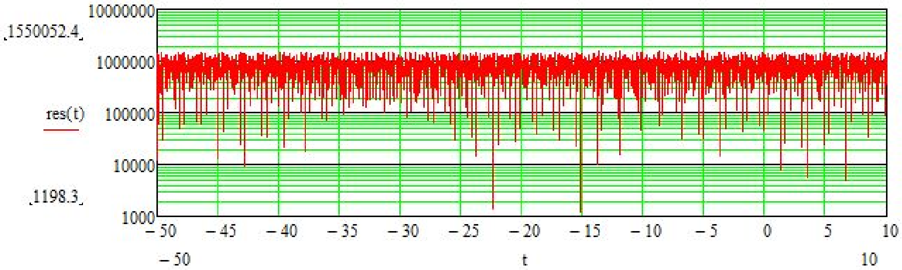

Fig. 2.4.1.b Logarithmic solar position vector module

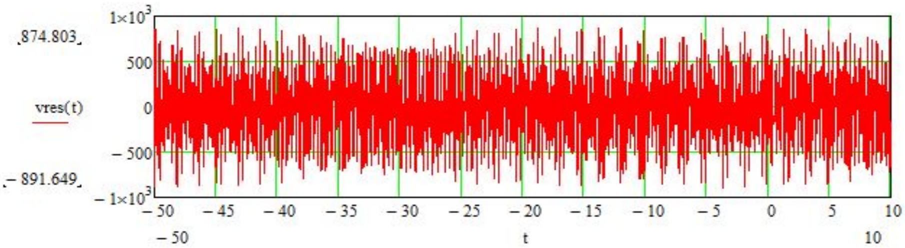

Fig. 2.4.2 Radial velocity of the Sun

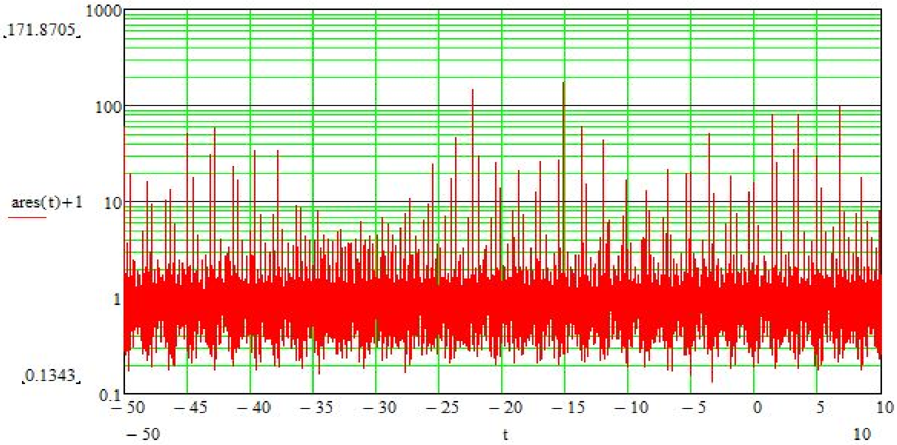

Fig. 2.4.3 Radial acceleration of the Sun

Comment 2.4.1: In fig. 2.4.3 in order to be able to use logarithmic coordinates for the Y axis (for a more suggestive representation) the negative values were avoided by the translation with a unit of ares(t).

3 - The individual contributions of the planets to the motion of the Sun[5]

Next we will analyse the individual contributions of the planets surrounding the Sun starting with the giant planets (which have the most important contribution) and continuing

[5] Without the corrections in table 1.5.1.B

with the telluric planets. As we will see below, the analysis of the individual contributions of the planets to the motion of the Sun is very important because on this basis we will be able to determine:

1. The number of spectral components (harmonics) produced by each planet, a number proportional to the eccentricity *e* of the orbit;
2. Ratio $q_{ai} = \dfrac{A(f_{1i})}{A(f_{2i})}$ where $A(f_{1i})$ and $A(f_{2i})$ are the amplitudes of the first and second harmonics produced by the planet *i* on solar motion, inversely proportional to *e*;
3. The lists of the individual spectral components of the planets will be the basis for calculating the positive frequency differences present in the spectrum of the solar orbital, these differences being the basis for estimating the weights of the contributions of different planets to the solar motion;
4. The global values of the solar parameters *velocity* and *orbital acceleration* are given by the relationships between these values and the values of the individual planetary contributions to these parameters (see comments 2.3.1 and 2.3.2).

Because the processes of solar motion are periodic, therefore characterized by a multitude of frequencies, we will establish some rules on the names adopted for these frequencies:

1. We assign each planet *i* ($i \in [0,7]$ [6]) a name *p* ($p \in [Me, Ve, Ea, Ma, Ju, Sa, Ur, Ne]$);
2. The *natural frequency* of the planet named *p* (fp0) is $f_i = 1/T_i$, where T_i is the revolution period of the planet *i* around the Sun, expressed in seconds;
3. The harmonics of the planet *p* (fp1...fpn) are the frequencies resulting from the spectral analysis of the contribution of planet *p* to solar motion.

3.1 – Jupiter's contribution to solar motion

The solar position due exclusively to Jupiter in ecliptic coordinates[7] is given by the equations:

$$xesJu(t) = -\frac{xe(4,t)}{q_4} \cdot AU \; ; \; yesJu(t) = -\frac{ye(4,t)}{q_4} \cdot AU \quad [km] \qquad (3.1.1)$$

and the graphical representation in Fig. 3.1.1:

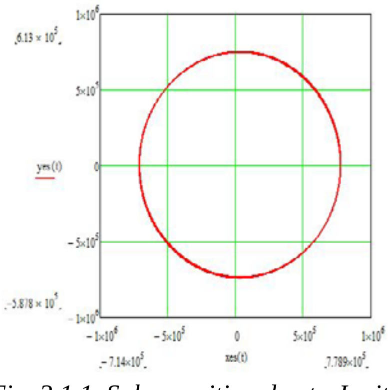

Fig. 3.1.1. Solar position due to Jupiter

First-order derived distribution of the solar position (solar velocity) due to Jupiter is given by the equations:

$$vxesJu(t) = \frac{xesJu(t) - xesJu(t-\Delta t)}{Nd} \; ; \; vyesJu(t) = \frac{yesJu(t) - yesJu(t-\Delta t)}{Nd} \left[{km}/{day} \right] \qquad (3.1.2)$$

---

[6] Pluto's contribution to solar motion is insignificant in amplitude, in addition, having a very long period of revolution, the spectral analysis over 60 centuries is inaccurate (there are only 24 periods).

[7] See Comment 1.5.1

where *xesJu(t)* and *yesJu(t)* are given by equations 3.1.1, and *vxesJu(t)* and *vyesJu(t)* are the components of the solar orbital velocity due to Jupiter and represented in Fig. 3.1.2:

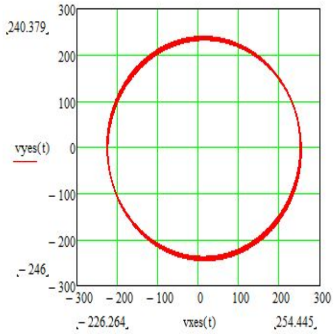

Fig. 3.1.2 Components of the solar orbital velocity due to Jupiter

The module of the solar orbital velocity due to Jupiter is given by the equation $vesJu(t) = \sqrt{vxesJu(t)^2 + vyesJu(t)^2}$ the representation of which is given in Fig. 3.1.3.

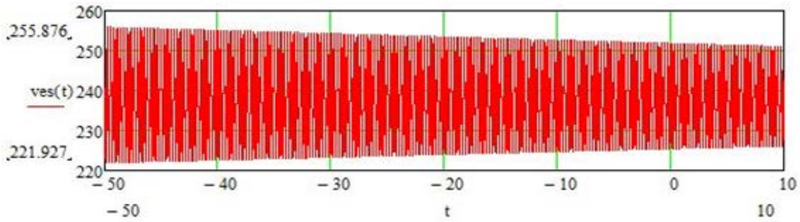

Fig. 3.1.3 Module of the solar orbital velocity due to Jupiter

Second-order derived distribution of the solar position due to Jupiter (solar orbital acceleration) is given by the equations:

$$axesJu(t) = \frac{vxesJu(t) - vxesJu(t - \Delta t)}{Nd}; ayesJu(t) = \frac{vyesJu(t) - vyesJu(t - \Delta t)}{Nd} \left[{km}/{day^2} \right] \quad (3.1.3)$$

where *vxesJu(t)* and *vyesJu(t)* are given by equations 3.1.2, and *axesJu(t)* and *ayesJu(t)* are the components of the solar orbital acceleration due to Jupiter and represented in Fig. 3.1.4:

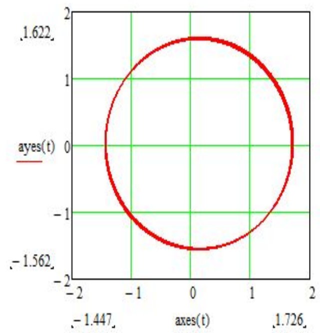

Fig. 3.1.4 Components of solar orbital acceleration due to Jupiter

The module of solar orbital acceleration $aesJu(t) = \sqrt{axesJu(t)^2 + ayesJu(t)^2}$ produced by Jupiter is shown in Fig. 3.1.5:

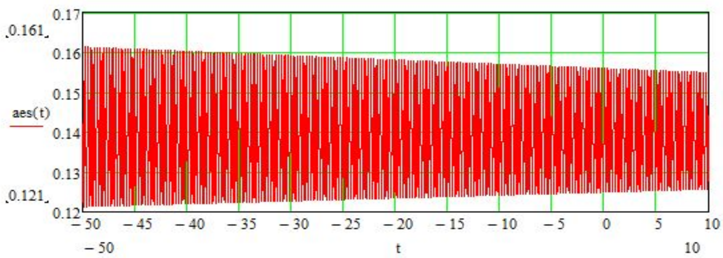

Fig. 3.1.5 Module of the solar orbital acceleration due to Jupiter

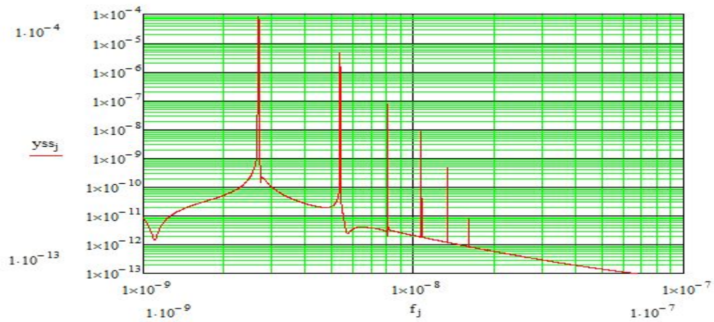

Fig. 3.1.6 aesJu pg spectrum

In Fig. 3.1.6 the parameter spectrum *aesJu(t)* from fig. 3.1.5 is represented following FFT analysis with data filtered through a Gaussian window type $pg(n) = e^{-\left(\frac{n-\mu}{\sigma}\right)^2}$; $\mu = \frac{N}{2}$; $\sigma = \frac{\mu}{3}$, where *n* is the sample number in the string of $N = 2^{18}$ samples.

The more important spectral components in terms of amplitude of the Jovian contribution to the solar *aes* resulting from Fig. 3.1.6 are given in table 3.1.1.

Table 3.1.1

Frequency [Hz]	Amplitude	Comments
f1=2.6724e-009	8.0105e-005	fJu1
f2=5.3447e-009	4.3270e-006	fJu2=2fJu1 q_a=18.51
f3=8.0118e-009	7.8814e-008	fJu3=3fJu1
f4=1.0684e-008	8.0121e-009	fJu4=4fJu1
f5=1.3357e-008	4.7326e-010	fJu5=5fJu1

3.2 – Saturn's contribution to solar motion

The solar position due exclusively to Saturn is given by the equations:

$$xesSa(t) = -\frac{xe(5,t)}{q_5} \cdot AU \; ; \; yesSa(t) = -\frac{ye(5,t)}{q_5} \cdot AU \quad [km] \qquad (3.2.1)$$

and their graphical representation is given in Fig. 3.2.1.

The derived distribution of the solar position due to Saturn is given by the equations:

$$vxesSa(t) = \frac{xesSa(t) - xesSa(t - \Delta t)}{Nd} \; ; vyesSa(t) = \frac{yesSa(t) - yesSa(t - \Delta t)}{Nd} \quad \left[{km}/{day} \right] \qquad (3.2.2)$$

where *xesSa(t)* şi *yesSa(t)* are given by equations 3.2.1, and *vxesSa(t)* and *vyesSa(t)* are the solar orbital velocities due to Saturn given by equations 3.2.2 and represented in Fig. 3.2.2.

The module of the solar orbital velocity due to Saturn is given by the equation $vesSa(t) = \sqrt{vxesSa(t)^2 + vyesSa(t)^2}$ the representation of which is given in Fig. 3.2.3:

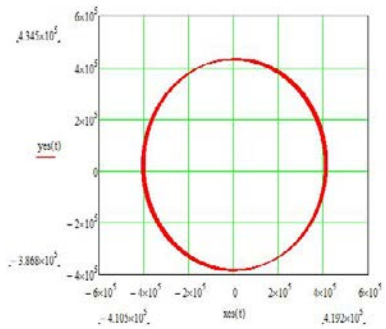

Fig. 3.2.1 Solar position due to Saturn

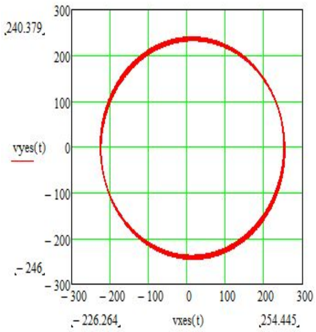

Fig. 3.2.2 Components of the solar orbital velocity due to Saturn

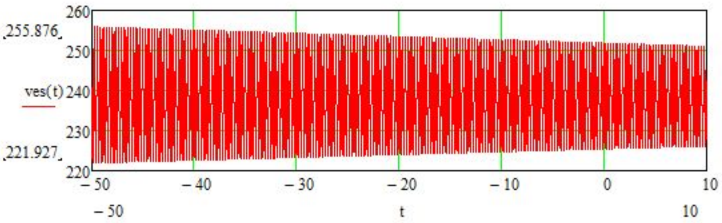

Fig. 3.2.3 Mdule of the solar orbital velocity due to Saturn

Second-order derived distribution of the solar position due to Saturn (solar orbital acceleration) is given by the equations:

$$axesSa(t) = \frac{vxesSa(t) - vxesSa(t-\Delta t)}{Nd}\,;\, ayesSa(t) = \frac{vyesSa(t) - vyesSa(t-\Delta t)}{Nd}\left[\frac{km}{day^2}\right] \qquad (3.2.3)$$

where *vxesSa(t)* and *vyesSa(t)* are given by equations 3.2.2, and *axesSa(t)* and *ayesSa(t)* are the components of the solar orbital acceleration due to Saturn and represented in Fig. 3.2.4:

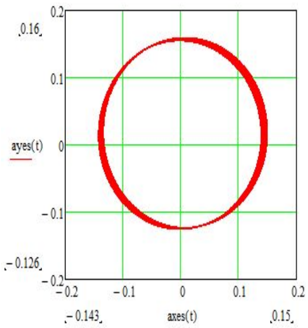

Fig. 3.2.4 Components of solar orbital acceleration due to Saturn

The module of solar orbital acceleration $aesSa(t) = \sqrt{axesSa(t)^2 + ayesSa(t)^2}$ produced by Saturn is shown in Fig. 3.2.5:

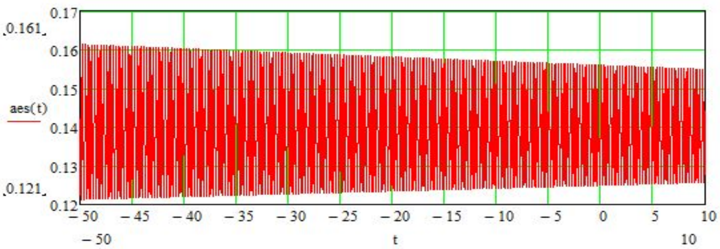

Fig. 3.2.5 Module of solar orbital acceleration due to Saturn

Spectral analysis of the parameter *aesSa(t)* shown in Fig. 3.2.5 is given in Fig. 3.2.6 under the same conditions as for Jupiter.

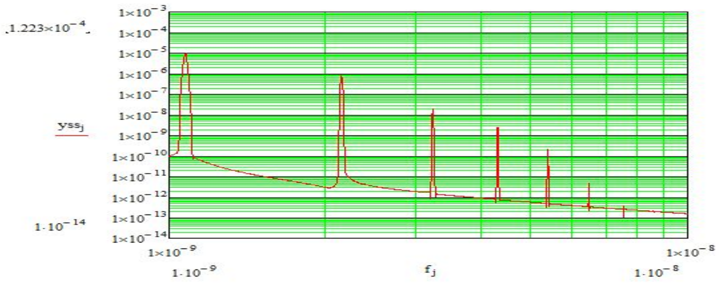

Fig. 3.2.5 aesSa pg spectrum

The spectral components of Saturn's contribution to the solar motion resulting from Fig. 3.2.5 are given in table 3.2.1.

Table 3.2.1

Frequency [Hz]	Amplitude	Comments
f1=1.0774e-009	9.5205e-006	fSa1
f2=2.1495e-009	7.6037e-007	fSa2=2fSa1 qSa=12.52
f3=3.2269e-009	1.8746e-008	fSa3=3fSa1
f4=4.299e-009	2.4874e-009	fSa4=4fSa1
f5=5.3764e-009	2.1108e-010	fSa5=5fSa1

3.3 - Uranus' contribution to solar motion

The solar position due exclusively to Uranus is given by the equations:

$$xesUr(t) = -\frac{xe(6,t)}{q_6}\cdot AU \; ; \; yesUr(t) = -\frac{ye(6,t)}{q_6}\cdot AU \quad [km] \qquad (3.3.1)$$

and their graphical representation is given in Fig. 3.3.1.

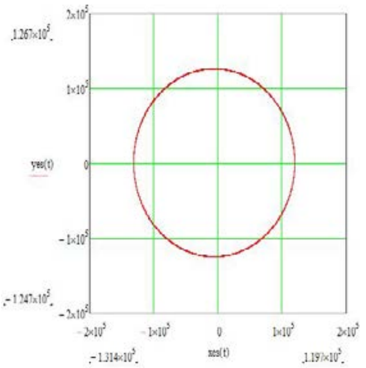

Fig. 3.3.1 Solar position due to Uranus

The derived distribution of the solar position due to Uranus is given by the equations:

$$vxesUr(t) = \frac{xesUr(t) - xesUr(t-\Delta t)}{Nd} \; ; \; vyesUr(t) = \frac{yesUr(t) - yesUr(t-\Delta t)}{Nd} \left[\frac{km}{day}\right] \qquad (3.3.2)$$

where *xesUr(t)* and *yesUr(t)* are given by equations 3.3.1, and *vxesUr(t)* and *vyesUr(t)* are the components of the solar orbital velocity due to Uranus given by equations 3.3.2 and represented in Fig. 3.3.2.

The module of the solar orbital velocity due to Uranus is given by the equation $vesUr(t) = \sqrt{vxesUr(t)^2 + vyesUr(t)^2}$ the representation of which is given in Fig. 3.3.3:

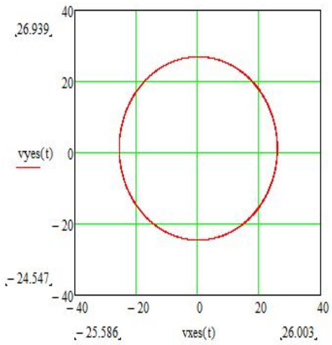

Fig. 3.3.2 Components of the solar orbital velocity due to Uranus

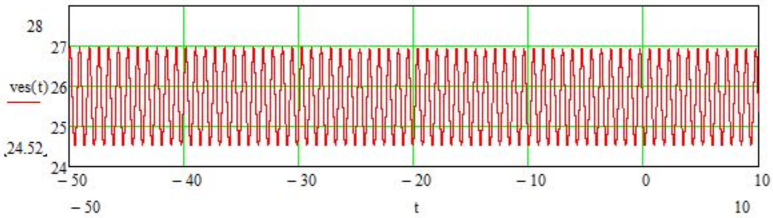

Fig. 3.3.3 Module of the solar orbital velocity due to Uranus

Second-order derived distribution of the solar position due to Uranus (solar orbital acceleration) is given by the equations:

$$axesUr(t) = \frac{vxesUr(t) - vxesUr(t-\Delta t)}{Nd}; \; ayesUr(t) = \frac{vyesUr(t) - vyesUr(t-\Delta t)}{Nd} \left[{km}/{day^2} \right] \quad (3.3.3)$$

where *vxesUr(t)* and *vyesUr(t)* are given by equations 3.3.2, and *axesUr(t)* and *ayesUr(t)* are the components of the solar orbital acceleration due to Uranus and represented in Fig. 3.3.4:

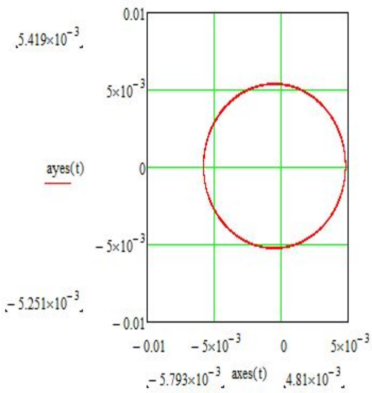

Fig. 3.3.4 Components of solar orbital acceleration due to Uranus

The module of solar orbital acceleration $aesUr(t) = \sqrt{axesUr(t)^2 + ayesUr(t)^2}$ produced by Uranus is represented in Fig. 3.3.5:

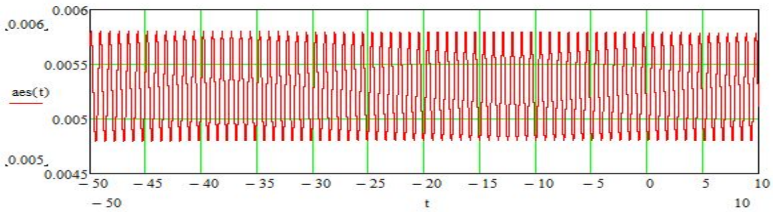

Fig. 3.3.5 Module of the solar orbital acceleration due to Uranus

Spectral analysis of the parameter *aesUr(t)* shown in Fig. 3.3.5 is given in Fig. 3.3.6 under the same conditions as for Jupiter and Saturn.

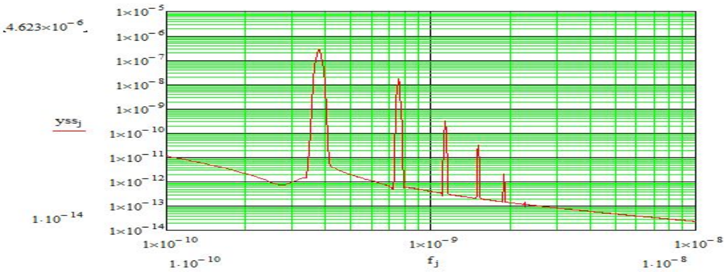

Fig. 3.3.6 aesUr pg spectrum

The spectral components of Uranus' contribution to the solar motion resulting from Fig. 3.3.6 are given in Table 3.3.1.

Table 3.3.1

Frequency	Amplitude	Comments
f1=3.7498e-010	2.744e-007	fUr1
f2=7.5523e-010	1.6678e-008	fUr2=2fUr1 qUr=16.45
f3=1.1302e-009	3.1394e-010	fUr3=3fUr1
f4=1.5105e-009	3.1352e-011	fUr4=4fUr1

3.4 – Neptune's contribution to solar motion

The solar position due exclusively to Neptune is given by the equations:

$$xesNe(t) = -\frac{xe(7,t)}{q_7} \cdot AU \ ; \ yesNe(t) = -\frac{ye(7,t)}{q_7} \cdot AU \quad [km] \qquad (3.4.1)$$

and their graphical representation is given in Fig. 3.4.1. First-order derived distribution of the solar position due to Neptune is given by the equations:

$$vxesNe(t) = \frac{xesNe(t) - xesNe(t-\Delta t)}{Nd} \ ; vyesNe(t) = \frac{yesNe(t) - yesNe(t-\Delta t)}{Nd} \left[{km}/{day} \right] \qquad (3.4.2)$$

where *xesNe(t)* and *yesNe(t)* are given by equations 3.4.1, and *vxesNe(t)* and *vyesNe(t)* are the solar orbital velocities due to Neptune, given by equations 3.4.2 and represented in Fig. 3.4.2.

The module of the solar orbital velocity due to Neptune is given by the equation $vesNe(t) = \sqrt{vxesNe(t)^2 + vyesNe(t)^2}$ the representation of which is given in Fig. 3.4.3:

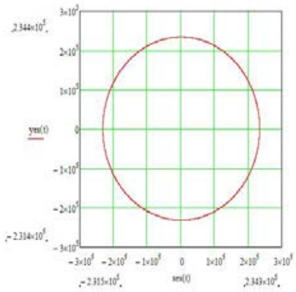

Fig. 3.4.1 Solar position due to Neptune

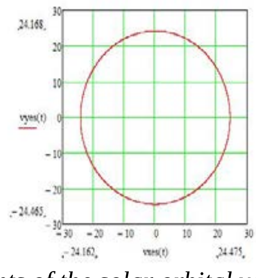

Fig. 3.4.2 Components of the solar orbital velocity due to Neptune

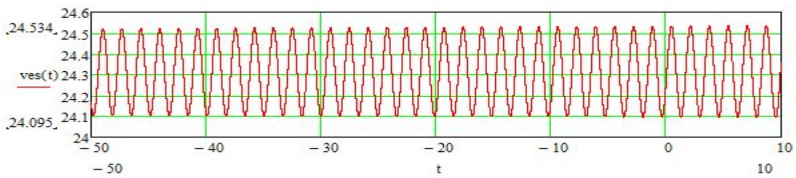

Fig. 3.4.3 Module of the solar orbital velocity due to Neptune

Second-order derived distribution of the solar position due to Neptune (solar orbital acceleration) is given by the equations:

$$axesNe(t)=\frac{vxesNe(t)-vxesNe(t-\Delta t)}{Nd};\ ayesNe(t)=\frac{vyesNe(t)-vyesNe(t-\Delta t)}{Nd}\left[{km}/{day^2}\right] \quad (3.4.3)$$

where *vxesNe(t)* and *vyesNe(t)* are given by equations 3.4.2, and *axesNe(t)* and *ayesNe(t)* are the components of the solar orbital acceleration due to Neptune and represented in fig. 3.4.4:

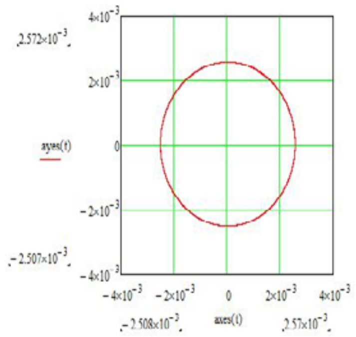

Fig. 3.4.4 Components of solar orbital acceleration due to Neptune

The module of solar orbital acceleration $aesNe(t)=\sqrt{axesNe(t)^2+ayesNe(t)^2}$ produced by Neptune is shown in Fig. 3.4.5:

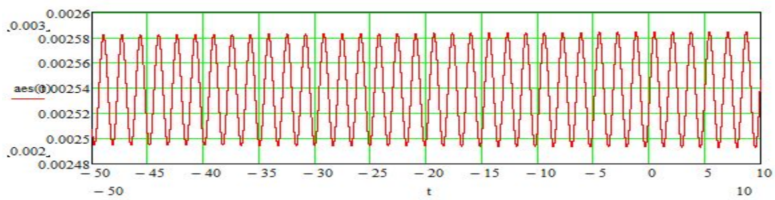

Fig. 3.4.5 Module of solar orbital acceleration due to Neptune

Spectral analysis of the parameter *aesNe(t)* shown in Fig. 3.4.5 is given in Fig. 3.4.6 under the same conditions as for Jupiter, Saturn and Uranus.

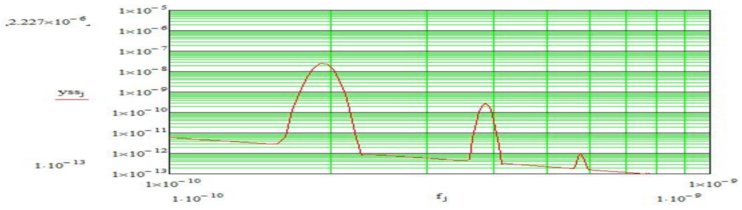

Fig. 3.4.6 aesNe pg spectrum

The spectral components of Neptune's contribution to the solar motion resulting from Fig. 3.4.6 are given in Table 3.4.1.

Table 3.4.1

Frequency	Amplitude	Comments
f1=1.9277e-010	2.3993e-008	fNe1
f2=3.8554e-010	2.8049e-010	fNe2=2fNe1 q_a=85.5
f3=5.7567e-010	1.013e-012	fNe3=3fNe1

3.5 - Mars' contribution to solar motion

The solar position due exclusively to Mars is given by the equations:

$$xesMa(t) = -\frac{xe(3,t)}{q_3} \cdot AU \; ; \; yesMa(t) = -\frac{ye(3,t)}{q_3} \cdot AU \quad [km] \qquad (3.5.1)$$

and their graphical representation is given in Fig. 3.5.1. First-order derived distribution of the solar position due to Mars is given by the equations:

$$vxesMa(t) = \frac{xesMa(t) - xesMa(t - \Delta t)}{Nd} \; ; \; vyesMa(t) = \frac{yesMa(t) - yesMa(t - \Delta t)}{Nd} \left[{km}/{day} \right] \qquad (3.5.2)$$

where *xesMa(t)* and *yesMa(t)* are given by equations 3.5.1, and *vxesMa(t)* and *vyesMa(t)* are the solar orbital velocities due to Mars, given by equations 3.5.2 and represented in Fig. 3.5.2.

The module of the solar orbital velocity due to Mars is given by the equation $vesMa(t) = \sqrt{vxesMa(t)^2 + vyesMa(t)^2}$ and does not show any variation in amplitude throughout the investigated duration, so we no longer represent it.

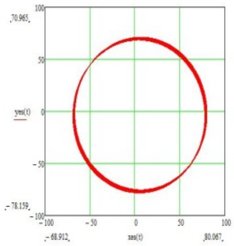

Fig. 3.5.1 Solar position due to Mars

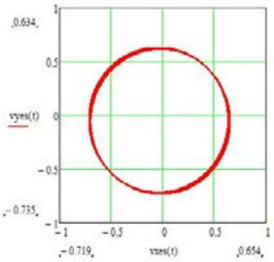

Fig. 3.5.2 Solar orbital velocity due to Mars

Second-order derived distribution of the solar position due to Mars (solar orbital acceleration) is given by the equations:

$$axesMa(t) = \frac{vxesMa(t) - vxesMa(t - \Delta t)}{Nd} \; ; \; ayesMa(t) = \frac{vyesMa(t) - vyesMa(t - \Delta t)}{Nd} \left[{km}/{day^2} \right] \qquad (3.5.3)$$

where *vxesMa(t)* and *vyesMa(t)* are given by equations 2.5.2, and *axesMa(t)* and *ayesMa(t)* are the components of the solar orbital acceleration due to Mars represented in Fig. 3.5.3:

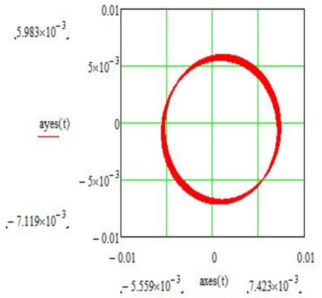

Fig. 3.5.3 Solar orbital acceleration due to Mars

The module of solar orbital acceleration $aesMa(t) = \sqrt{axesMa(t)^2 + ayesMa(t)^2}$ produced by Mars shows no variation in time so we no longer represent it, but its spectral analysis is given in Fig. 3.5.4 under the same conditions as for giant planets.

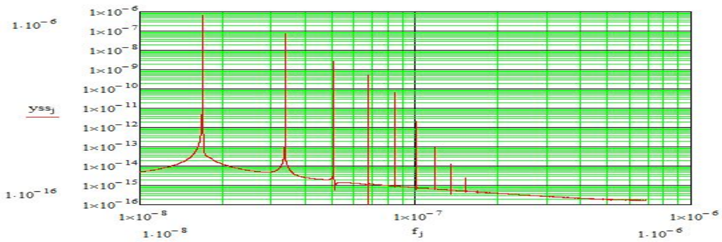

Fig. 3.5.4 aesMa pg spectrum

The spectral components of Mars' contribution to the solar motion resulting from Fig. 3.5.4 are given in Table 3.5.1.

Table 3.5.1

Frequency	Amplitude	Comments
f1=1.68475e-008	6.6435e-007	fMa1
f2=3.3695e-008	7.6489e-008	fMa2=2fMa1 qMa=8.85
f3=5.05425e-008	2.8241e-009	fMa3=3fMa1
f4=6.739e-008	5.8027e-010	fMa4=4fMa1
f5=8.42375e-008	7.0792e-011	fMa5=5fMa1

3.6 - Earth's contribution to solar motion

The solar position due exclusively to the Earth is given by the equations:

$$xesEa(t) = -\frac{xe(2,t)}{q_2} \cdot AU \; ; \; yesEa(t) = -\frac{ye(2,t)}{q_2} \cdot AU \quad [km] \tag{3.6.1}$$

and heir graphical representation is given in Fig. 3.6.1. First-order derived distribution of the solar position due to the Earth is given by the equations:

$$vxesEa(t) = \frac{xesEa(t) - xesEa(t-\Delta t)}{Nd} \; ; \; vyesEa(t) = \frac{yesEa(t) - yesEa(t-\Delta t)}{Nd} \quad \left[{}^{km}\!/\!{}_{day} \right] \tag{3.6.2}$$

where *xesEa(t)* and *yesEa(t)* are given by equations 3.6.1, and *vxesEa(t)* and *vyesEa(t)* are the components of the solar orbital velocity due to the Earth, given by equations 3.6.2 and represented in Fig. 3.6.2. Second-order derived distribution of the solar position due to the Earth (solar orbital acceleration) is given by the equations:

$$axesEa(t) = \frac{vxesEa(t) - vxesEa(t-\Delta t)}{Nd} \; ; \; ayesEa(t) = \frac{vyesEa(t) - vyesEa(t-\Delta t)}{Nd} \left[{}^{km}\!/\!{}_{day^2} \right] \tag{3.6.3}$$

where *vxesEa(t)* and *vyesEa(t)* are given by equations 3.6.2, and *axesEa(t)* and *ayesEa(t)* are the components of the solar orbital acceleration due to the Earth represented in Fig. 3.6.3:

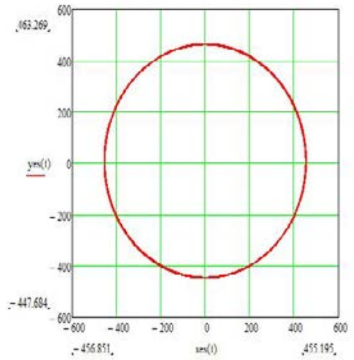

Fig. 3.6.1 Solar position due to the Earth

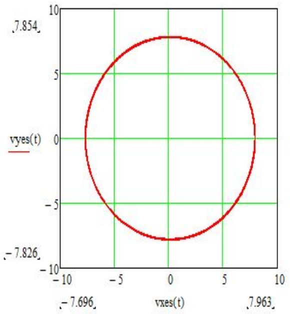

Fig. 3.6.2 Solar orbital velocity due to Earth

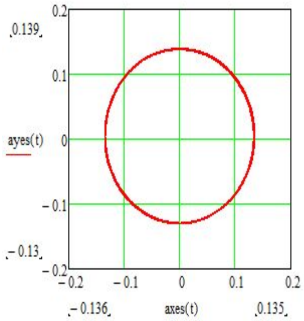

Fig. 3.6.3 Solar orbital acceleration due to Earth

The module of solar orbital acceleration is $aesEa(t) = \sqrt{axesEa(t)^2 + ayesEa(t)^2}$ the spectrum of which is given in Fig. 3.6.4:

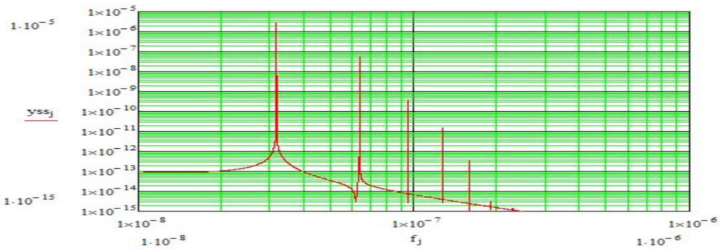

Fig. 2.6.4 aesEa pg spectrum

The spectral components of Earth’s contribution to solar orbital acceleration *aesEa(t)*, are given in Table 3.6.1.

Table 3.6.1

Frequency	Amplitude	Comments
f1=3.1688e-008	2.6756e-006	fEa1
f2=6.3376e-008	5.6677e-008	fEa2=2fEa1 qEa=47.21
f3=9.5064e-008	3.8158e-010	fEa3=3fEa1
f4=1.2675e-007	1.4964e-011	fEa4=4fEa1
f5=1.5844e-007	3.4538e-013	fEa5=5fEa1

3.7 - Venus' contribution to solar motion

The solar position due exclusively to Venus is given by the equations:

$$xesVe(t) = -\frac{xe(1,t)}{q_1} \cdot AU \;;\; yesVe(t) = -\frac{ye(1,t)}{q_1} \cdot AU \quad [km] \qquad (3.7.1)$$

and their graphical representation is given in Fig. 3.7.1. First-order derived distribution of the solar position due to Venus is given by the equations:

$$vxesVe(t) = \frac{xesVe(t) - xesVe(t - \Delta t)}{Nd} \;;\; vyesTe(t) = \frac{yesVe(t) - yesVe(t - \Delta t)}{Nd} \left[{km}/{day} \right] \qquad (3.7.2)$$

where *xesVe(t)* and *yesVe(t)* are given by equations 3.7.1, and *vxesVe(t)* and *vyesVe(t)* are the components of the solar orbital velocity due to Venus, given by equations 3.7.2 and represented in Fig. 3.7.2.

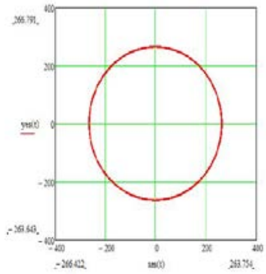

Fig. 3.7.1 Solar position due to Venus

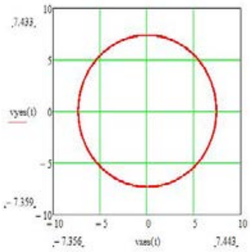

Fig. 3.7.2 Solar orbital velocity due to Venus

Second-order derived distribution of the solar position due to Venus (solar orbital acceleration) is given by the equations:

$$axesVe(t) = \frac{vxesVe(t) - vxesVe(t - \Delta t)}{Nd}\,;\, ayesVe(t) = \frac{vyesVe(t) - vyesVe(t - \Delta t)}{Nd}\left[{km}/{day^2}\right] \quad (3.7.3)$$

where *vxesVe(t)* and *vyesVe(t)* are given by equations 3.7.2, and *axesVe(t)* and *ayesVe(t)* are the components of the solar orbital acceleration due to Venus represented in Fig. 3.7.3:

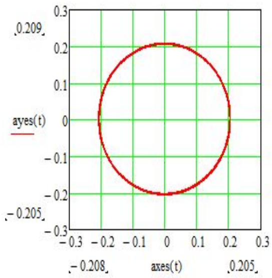

Fig. 3.7.3 Solar orbital acceleration due to Venus

The module of solar orbital acceleration is $aesVe(t) = \sqrt{axesVe(t)^2 + ayesVe(t)^2}$ the spectrum of which is given in Fig. 3.7.4:

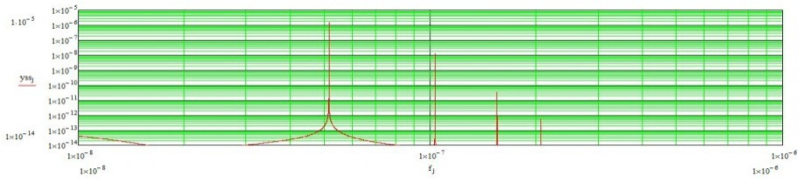

Fig. 3.7.4 aesVe pg spectrum

The spectral components of Venus' contribution to solar orbital acceleration *aesVe(t)*, are given in Table 3.7.1.

Table 3.7.1

Frequency	Amplitude	Comments
f1=5.1509e-008	1.8261e-006	fVe1
f2=1.0302e-007	1.7348e-008	fVe2=2fVe1 qVe=105.26
f3=1.5453e-007	5.3764e-011	fVe3=3fVe1
f4=2.0604e-007	9.2124e-013	fVe4=4fVe1

3.8 - Mercury's contribution to solar motion

The solar position due exclusively to Mercury is given by the equations:

$$xesMe(t) = -\frac{xe(0,t)}{q_0} \cdot AU \; ; \; yesMe(t) = -\frac{ye(0,t)}{q_0} \cdot AU \quad [km] \qquad (3.8.1)$$

and their graphical representation is given in Fig. 3.8.1. First-order derived distribution of the solar position due to Mercury is given by the equations:

$$vxesMe(t) = \frac{xesMe(t) - xesMe(t-\Delta t)}{Nd} \; ; \; vyesMe(t) = \frac{yesMe(t) - yesMe(t-\Delta t)}{Nd} \left[{km}/{day} \right] \qquad (3.8.2)$$

where *xesMe(t)* and *yesMe(t)* are given by equations 3.8.1, and *vxesMe(t)* and *vyesMe(t)* are the components of the solar orbital velocity due to Mercury, given by equations 3.8.2 and represented in Fig. 3.8.2.

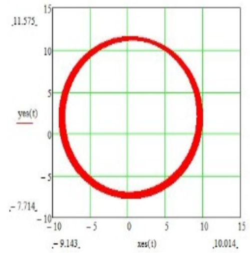

Fig. 3.8.1 Solar position due to Mercury

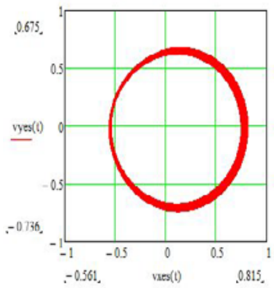

Fig. 3.8.2 Solar orbital velocity due to Mercury

Second-order derived distribution of the solar position due to Mercury (solar orbital acceleration) is given by the equations:

$$axesMe(t) = \frac{vxesMe(t) - vxesMe(t-\Delta t)}{Nd} \; ; \; ayesMe(t) = \frac{vyesMe(t) - vyesMe(t-\Delta t)}{Nd} \left[{km}/{day^2} \right] \qquad (3.8.3)$$

where *vxesMe(t)* and *vyesMe(t)* are given by equations 3.8.2, and *axesMe(t)* and *ayesMe(t)* are the components of the solar orbital acceleration due to Mercury represented in Fig. 3.8.3.

The module of solar orbital acceleration is $aesMe(t) = \sqrt{axesMe(t)^2 + ayesMe(t)^2}$ the spectrum of which is given in Fig. 3.8.4.

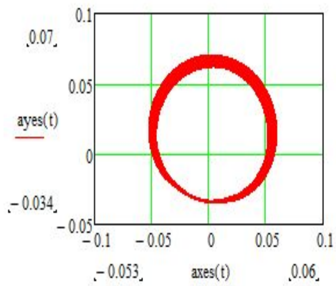

Fig. 3.8.3 Solar orbital acceleration due to Mercury

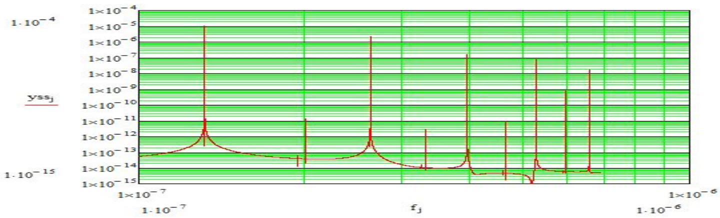

Fig. 3.8.4 aesMe pg spectrum

The spectral components of Mercury's contribution to solar orbital acceleration *aesMe(t)*, are given in Table 3.8.1.

Table 3.8.1

Frequency	Amplitude	Comments
f1=1.3157e-007	1.0364e-005	fMe1
fa=2.0035e-007	1.3783e-011	fMe2-fEa2
f2=2.6314e-007	2.1445e-006	fMe2=2fMe1 qMe=4.83
fb=3.3192e-007	2.7668e-012	fMe3-fEa2
f3=3.9471e-007	1.6356e-007	fMe3=3fMe1
fc=4.6349e-007	9.0956e-012	fMe4-fEa2
f4=5.2628e-007	7.5607e-008	fMe4=4fMe1
fd=5.9506e-007	8.0831e-010	fMe5-fEa2
f5=6.5784e-007	1.7709e-008	fMe5=5fMe1

Comment 3.8.1: From the table above we can observe both the spectral components due to the ellipticity of the Mercury orbit (components fMe1 ... fMe5), and the components fa ... fd due to the influence of the Earth.

4 - The amplitude of the spectral components depends on the eccentricity of the orbit

Let us summarize the spectral components of the planetary contributions to solar motion, specifying the eccentricity of the orbit and adding the orbital frequencies[8] with the suffix 0.

Table 4.1 - Neptune, e=0.00895439

Name	Frequency [Hz]	Amplitude [A]	Comments
fNe0	1.92295E-10		
fNe1	1.9277e-010	2.3993e-008	
fNe2	3.8554e-010	2.8049e-010	2fNe1 q_{a7}=85.5
fNe3	5.7567e-010	1.013e-012	3fNe1

Table 4.2 - Uranus, e=0.04716771

Name	Frequency [Hz]	Amplitude [A]	Comments
fUr0	3.771850E-10		
fUr1	3.7498e-010	2.744e-007	
fUr2	7.5523e-010	1.6678e-008	2fUr1 q_{a6}=16.45
fUr3	1.1302e-009	3.1394e-010	3fUr1
fUr4	1.5105e-009	3.1352e-011	4fUr1

Table 4.3 - Saturn, e=0.05415060

Name	Frequency [Hz]	Amplitude [A]	Comments
fSa0	1.07574E-09		
fSa1	1.0774e-09	9.5205e-006	
fSa2	2.1495e-09	7.6037e-007	2fSa1 q_{a5}=12.52
fSa3	3.2269e-09	1.8746e-008	3fSa1
fSa4	4.299e-09	2.4874e-009	4fSa1
fSa5	5.3764e-09	2.1108e-010	5fSa1

Table 4.4 - Jupiter, e=0.04839266

Name	Frequency [Hz]	Amplitude [A]	Comments
fJu0	2.6714E-09		
fJu1	2.6724e-09	8.0105e-005	
fJu2	5.3447e-09	4.327e-006	2fJu1 q_{a4}=18.5
fJu3	8.0118e-09	7.8814e-008	3fJu1

[8] The planetary orbital frequency (also called natural planetary frequency) in Hz is the inverse of the planetary period expressed in seconds.

fJu4	1.0684e-08	8.0121e-009	4fJu1
fJu5	1.3357e-08	4.7326e-010	5fJu1

Table 4.5 - Mars, e=0.09341233

Name	Frequency [Hz]	Amplitude [A]	Comments
fMa0	1.684776E-08		
fMa1	1.68475E-08	6.6435e-07	
fMa2	3.3695E-08	7.6489e-08	2fMa1 q_{a3}=8.85
fMa3	5.054250E-08	2.8241e-09	3Ma1
fMa4	6.739E-08	5.8027e-10	4fMa1
fMa5	8.423750E-08	7.0792e-11	5fMa1

Table 4.6 - Earth, e=0.01671022

Name	Frequency [Hz]	Amplitude [A]	Comments
fEa0	3.168757E-08		
fEa1	3.16881E-08	2.6756e-06	
fEa2	6.33762E-08	5.6677e-08	2fEa1 q_{a2}=47.21
fEa3	9.50643E-08	3.8158e-10	3fEa1
fEa4	1.26752E-07	1.4964e-11	4fEa1
fEa5	1.58435E-07	3.4538e-13	5fEa1

Table 4.7 - Venus, e=0.00677323

Name	Frequency [Hz]	Amplitude [A]	Comments
fVe0	5.150878E-08		
fVe1	5.1509E-08	1.8261e-06	
fVe2	1.03018E-07	1.7348e-08	2fVe1 q_{a1}=105.26
fVe3	1.54527E-07	5.3764e-11	3fVe1
fVe4	2.06036E-07	9.2124e-13	4fVe1

Table 4.8 - Mercury, e=0.20563069

Name	Frequency [Hz]	Amplitude [A]	Comments
fMe0	1.315699E-07		
fMe1	1.3157e-07	1.0364e-05	
fMe2	2.63143E-07	2.1445e-06	2fMe1 q_{a0}=4.82
fMe3	3.94707e-07	1.6452e-07	3fMe1
fMe4	5.26281e-07	7.6224e-08	4fMe1
fMe5	6.57855e-07	1.7893e-08	5fMe1

Comment 4.1: The data in tables 4.1 ... 4.8 are the basis for calculating the positive frequency differences that we will use to identify the components resulting from the spectral analysis, components that we will find in Table 5.1. With the data from the above tables, 8 matrices are constructed with which the positive differences of two terms from different matrices are calculated. The result is an Excel file with 730 possible components, of which the 190 frequencies in table 5.1 can be found.

From tables 4.1 ... 4.8 we can note that there is a direct dependence relation between the value of the eccentricity *e* of the orbit of a given planet and the number of harmonics[9] of the contribution of that planet to the solar motion. There is also (see Table 4.9) an inverse dependence relationship between the value of the eccentricity *e* (in ascending order in the table) and the ratio of the amplitudes of the first two spectral components q_{ai} .

Table 4.9

Eccentricity *e*	Planet	q_{ai}	Comment
0.00677323	Venus	105.26	
0.00895439	Neptune	85.5	
0.01671022	Earth	47.21	
0.04716771	Uranus	16.45	Uranus anomaly [10]
0.04839266	Jupiter	18.5	
0.0541506	Saturn	12,52	
0.09341233	Mars	8.85	
0.20563069	Mercury	4.82	

[9] We are talking about harmonics with important amplitudes.

[10] According to the value of the eccentricity the value q_{ai} for Uranus should be between 47 and 18. The reason why it turned out to be a very different value is not known.

5 - The spectral components of the solar *aes* parameter over a time interval of 60 centuries for all planets.

Next we will analyse the spectral components of the solar orbital acceleration over the time interval of the validity of the ephemerides table (60 centuries), without the data from Table 1.5.1.B. The *aes* spectrum under these conditions is given in Fig. 5.1:

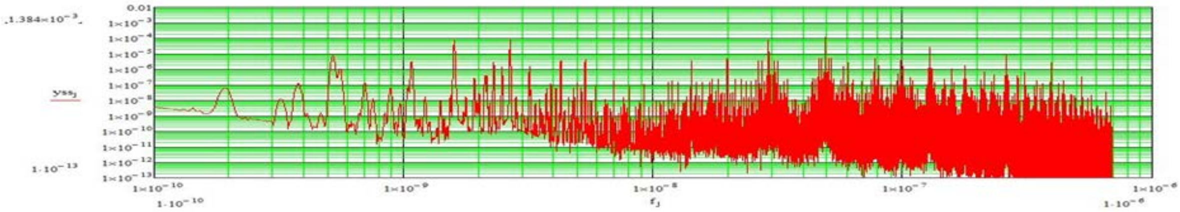

Fig. 5.1 Aes spectrum for all planets

For a more detailed image the same spectrum, but expanded in Fig. 5.2, 5.3 and 5.4, it is worth observing the frequency ranges at the bottom of the spectrum.

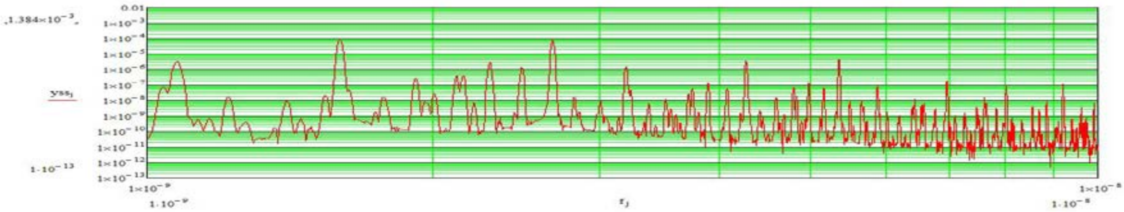

Fig. 5.2 Aes spectrum for all planets zoom 1

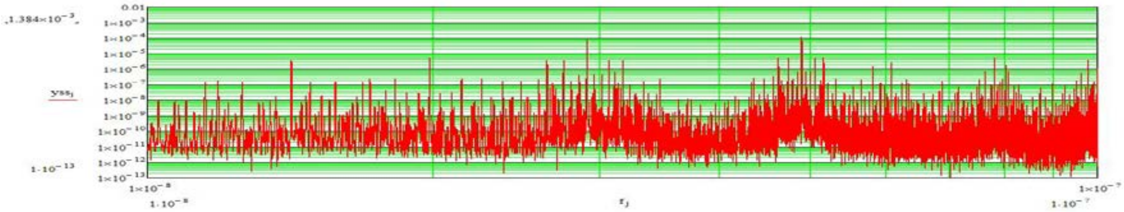

Fig. 5.3 Aes spectrum for all planets zoom 2

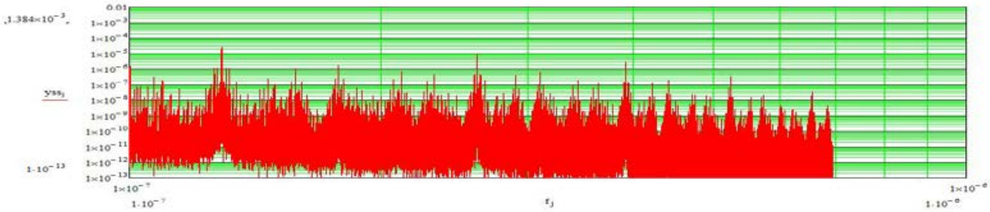

Fig. 5.4 Aes spectrum for all planets zoom 3

The important spectral components in terms of amplitude identified in the spectrum of Fig. 5.1 are given in Table 5.1, in which the ellipsis (...) indicate an interval with unidentified components. In column 4 (components) the name of the harmonic or the positive difference of harmonics of the planetary contributions to the solar parameter *aes* are entered, and in column 5 the name of the formant which the respective frequency is part of.

The amplitude of the components does not have units of measurement, it being useful only when comparing the values between them. Spectral analysis was also done by filtering the data through a Gaussian window.

Table 5.1

No.	Frequency [Hz]	Amplitude	Components	Formant[11]
f01	1.9277E-10	6.0317E-08	fNe1	
f02	3.2216E-10	1.3196E-08	fSa1-fUr2	
f03	3.7498E-10	1.2779E-07	fUr1	

[11] Regarding the notation of the formants, see par. 6.2

f04	3.8554E-10	6.5233E-08	fNe2	
f05	4.3307E-10	2.1454E-10	fUr4-fSa1	
f06	5.0173E-10	3.2384E-07	fSa1-fNe3	
f07	5.2285E-10	9.3117E-06	fJu1-fSa2	
f08	5.5454E-10	1.0426E-06	fUr3-fNe3	
f09	5.7567E-10	1.5136E-08	fNe3	
f10	6.3904E-10	1.5507E-10	fSa2-fUr4	
f11	6.9186E-10	8.7879E-08	fSa1-fNe2	
f12	7.0242E-10	1.1846E-07	fSa1-fUr1	
f13	7.5523E-10	1.2400E-08	fUr2	
f14	8.8199E-10	6.4734E-08	fSa0-fNe1	
f15	9.3480E-10	4.0132E-10	fUr4-fNe3	
f16	9.6649E-10	6.6169E-09	fVe1-fMa3	
f17	1.0404E-09	7.2839E-08	fJu2-fSa4	
f18	1.0774E-09	3.2848E-06	fSa1	
f19	1.1302E-09	6.3364E-10	fUr3	
f20	1.1619E-09	7.1672E-10	fJu0-fUr4	
f21	1.3151E-09	3.7548E-11	fUr4-fNe1	
f22	1.3943E-09	5.0370E-09	fSa2-fUr2	
f23	1.5105E-09	4.8875E-11	fUr4	
f24	1.5422E-09	2.1305E-08	fJu0-fUr3	
f25	1.5950E-09	7.8977E-05	fJu1-fSa1	GAf1
f26	1.7745E-09	1.6899E-08	fSa2-fUr1	
f27	1.9171E-09	2.8213E-07	fJu0-fUr2	
f28	1.9594E-09	8.3375E-09	fSa2-fNe1	
f29	2.1178E-09	4.0970E-07	fJu2-fSa3	
f30	2.1495E-09	4.1631E-07	fSa2	
f31	2.2921E-09	2.8750E-06	fJu1-fUr1	GAf2
f32	2.4770E-09	1.3846E-06	fJu1-fNe1	GAf3
f33	2.6724E-09	7.9417E-05	fJu1	GAf4
f34	2.8466E-09	1.7795E-09	fSa3-fUr0	
f35	3.0315E-09	9.0140E-10	fSa3-fNe1	
f36	3.1899E-09	1.6129E-06	fJu2-fSa2	GBf1
f37	3.8924E-09	1.2347E-07	fJu2-fUr1-fSa1	GBf2
f38	4.0772E-09	5.9679E-08	fJu2-fNe1-fSa1	GBf3
f39	4.2092E-09	1.4211E-09	fJu2-fUr3	
f40	4.2673E-09	3.6127E-06	fJu2-fSa1	GBf4, GCf1
f41	4.5895E-09	1.5827E-08	fJu2-fUr2	
f42	4.7849E-09	6.7322E-08	fJu3-fSa3	
f43	4.9645E-09	1.3612E-07	fJu2-fUr1	GCf2
f44	5.1493E-09	6.5549E-08	fJu2-fNe1	GCf3
f45	5.3447E-09	4.3231E-06	fJu2	GCf4
f46	5.8623E-09	7.7154E-08	fJu3-fSa2	GDf1

f47	6.4168E-09	1.1821E-08	fJu4-fSa4	
f48	6.5594E-09	6.0308E-09	fJu3-fUr1-fSa1	GDf2
f49	6.7443E-09	2.8969E-09	fJu3-fNe1-fSa1	GDf3
f50	6.9397E-09	1.7843E-07	fJu3-fSa1	GDf4, GEf1
f51	7.2407E-09	1.5209E-09	fJu3-fUr2	
f52	7.4573E-09	3.5003E-09		
f53	7.5999E-09	1.5159E-08	fJu3-fUr1	GEf2
f54	7.6368E-09	6.9195E-09		
f55	7.8217E-09	3.3276E-09	fJu3-fNe1	GEf3
f56	8.0118E-09	8.0243E-08	fJu3	GEf4, GFf1
f57	8.8357E-09	7.8850E-09	fMa1-fJu3	
f58	9.1948E-09	1.1349E-07	fJu4-fUr4	GFf4
f59	9.6068E-09	3.6250E-09	fJu4-fUr3	
f60	9.9078E-09	6.9750E-09	fJu4-fUr2	GGf1
f61	1.0103E-08	1.0103E-08	fJu4-fNe3	
f62	1.0298E-08	1.0272E-08	fJu4-fNe2	GGf2
f63	1.0462E-08	2.6156E-10	fJu4-fNe1	
f64	1.0790E-08	5.3963E-09	fJu4	GGf3
f65	1.1503E-08	1.5134E-07	fMa1-fJu2	Maf1
f66	1.2580E-08	1.5214E-07	fMa1-fSa4	Maf2
f67	1.2834E-08	1.9226E-10	fEa2-fMa3	
f68	1.3658E-08	1.9184E-08	fMa1-fSa3	
f69	1.4175E-08	3.4446E-06	fMa1-fJu1	Maf3
f70	1.4698E-08	1.9530E-08	fMa1-fSa2	
f71	1.4841E-08	1.5252E-07	fEa1-fMa1	
f72	1.5358E-08	8.6682E-10	fMa1-fUr4	
f73	1.5770E-08	1.5310E-07	fMa1-fSa1	Maf4
f74	1.6076E-08	1.1072E-09	fMa1-fUr2	
f75	1.6219E-08	1.9206E-09	fMa1-fNe3	
f76	1.6436E-08	6.9420E-09	fMa1-fUr1	
f77	1.6631E-08	1.6098E-08	fMa1-fNe1	
f78	1.6848E-08	1.5371E-07	fMa1	Maf5
f79	1.7814E-08	4.2939E-08	fVe1-fMa2	
f80	1.9821E-08	5.1482E-06	fVe1-fEa1	
f81	2.0898E-08	3.1139E-08	fEa1-fJu4	
f82	2.3011E-08	1.5747E-08	fMa2-fJu4	
f83	2.3676E-08	1.6513E-07	fEa1-fJu3	
f84	2.5683E-08	1.0670E-09	fMa2-fJu3	
f85	2.6343E-08	3.4216E-06	fEa1-fJu2	Eaf1
f86	2.7421E-08	3.3538E-06	fEa1-fSa4	Eaf2
f87	2.7796E-08	3.6026E-09	fVe3-fEa4	
f88	2.8498E-08	4.0884E-07	fEa1-fSa3	
f89	2.9016E-08	7.6631E-05	fEa1-fJu1	Eaf3

f90	2.9401E-08	8.8963E-09	fMa2-fSa4	
f91	2.9539E-08	4.1671E-07	fEa1-fSa2	
f92	3.0611E-08	3.4374E-06	fEa1-fSa1	Eaf4
f93	3.1023E-08	6.3130E-07	fMa2-fJu1	
f94	3.1308E-08	1.2393E-07	fEa1-fUr1	
f95	3.1688E-08	3.3787E-06	fEa1	Eaf5
f96	3.2206E-08	8.4673E-08	fMa2-fUr4	
f97	3.2618E-08	2.8110E-08	fMa2-fSa1	
f98	3.3695E-08	2.8194E-08	fMa2	
f99	3.4661E-08	2.3486E-07	fVe1-fMa1	
f100	3.6505E-08	1.1237E-09	fMe1-fEa3	
f101	3.9642E-08	6.1020E-09	fVe2-fEa2	
f102	4.0883E-08	7.8851E-09	fVe1-fJu4	
f103	4.3497E-08	2.5799E-07	fVe1-fJu3	
f104	4.4569E-08	2.3679E-07	fEa3-fMa3	
f105	4.6164E-08	5.1759E-06	fVe1-fJu2	Vef1
f106	4.6545E-08	1.9189E-07	fEa2-fMa1	
f107	4.7242E-08	5.1709E-06	fVe1-fSa4	Vef2
f108	4.7759E-08	2.3682E-07	fMa3-fJu1	
f109	4.8319E-08	6.4081E-07	fMa3-fSa2	
f110	4.8837E-08	1.1721E-04	fVe1-fJu1	Vef3
f111	4.9359E-08	6.4764E-07	fVe1-fSa2	
f112	4.9914E-08	6.7901E-07	fVe1-fUr4	
f113	5.0432E-08	5.2159E-06	fVe1-fSa1	Vef4
f114	5.1134E-08	1.9128E-07	fVe1-fUr1	
f115	5.1509E-08	5.2981E-06	fVe1	Vef5
f116	5.5359E-08	7.3247E-08	fEa2-fJu3	
f117	5.6436E-08	2.1264E-07	fMa4-fJu4	
f118	5.8031E-08	1.6051E-06	fEa2-fJu2	Ea2f1
f119	5.9109E-08	1.2620E-07	fEa2-fSa4	Ea2f2
f120	6.0704E-08	2.6841E-06	fEa2-fJu1	Ea2f3
f121	6.2299E-08	1.2204E-07	fEa2-fSa1	Ea2f4
f122	6.3012E-08	2.1797E-07	fEa2-fUr1	
f123	6.3012E-08	2.1797E-07	fMa4-fSa4	
f124	6.3376E-08	1.1592E-07	fEa2	Ea2f5
f125	6.4718E-08	6.1990E-09	fMa4-fJu1	
f126	6.7063E-08	2.3101E-08	fMa4-fUr1	
f127	6.8193E-08	3.9538E-08	fMe1-fEa2	
f128	7.1330E-08	7.0599E-08	fVe2-fEa1	
f129	7.6257E-08	6.5869E-07	fEa4-fMa3	
f130	8.0060E-08	1.7808E-06	fMe1-fVe1	
f131	8.7047E-08	6.6993E-08	fEa3-fJu3	
f132	8.9720E-08	1.1483E-07	fEa3-fJu2	Ea3f1

f133	9.0433E-08	6.1926E-09	fEa3-fSa4	Ea3f2
f134	9.1315E-08	5.3967E-09	fVe3-fEa2	
f135	9.2392E-08	7.7980E-08	fEa3-fJu1	Ea3f3
f136	9.3406E-08	2.3068E-08	fEa3-fUr4	
f137	9.4483E-08	5.4836E-08	fEa3-fNe3	
f138	9.5006E-08	1.6941E-07	fVe2-fJu3	
…				
f139	9.7673E-08	3.7820E-06	fVe2-fJu2	Ve2f1
f140	1.0035E-07	1.6052E-06	fVe2-fJu1	Ve2f3
f141	1.0194E-07	7.1565E-08	fVe2-fSa1	Ve2f4
f142	1.0302E-07	7.3361E-08	fVe2	Ve2f5
f143	1.0954E-07	1.9129E-07	fEa4-fMa1	
f144	1.1472E-07	5.3270E-08	fMe1-fMa1	
f145	1.1606E-07	3.4905E-09	fEa4-fJu4	
f146	1.1874E-07	7.2916E-09	fEa4-fJu3	
f147	1.2141E-07	5.4194E-09	fEa4-fJu2	
f149	1.2243E-07	4.7765E-09	fEa4-fSa4	
f150	1.2356E-07	5.9976E-08	fMe1-fJu3	
f151	1.2463E-07	5.2540E-08	fEa4-fSa2	
f152	1.2623E-07	1.1361E-06	fMe1-fJu2	Mef1
f153	1.2730E-07	1.1528E-06	fMe1-fSa4	Mef2
f154	1.2890E-07	2.5984E-05	fMe1-fJu1	Mef3
f155	1.3049E-07	1.1503E-06	fMe1-fSa1	Mef4
f156	1.3157E-07	1.1064E-06	fMe1	Mef5
f157	1.4651E-07	2.3864E-07	fVe3-fJu3	
f158	1.4918E-07	1.0009E-07	fVe3-fJu2	
f159	1.5185E-07	1.6223E-08	fVe3-fJu1	
f160	1.5411E-07	1.5694E-08	fVe3-fNe2	
f161	1.6058E-07	1.4572E-07	fMe2-fVe2	
f162	1.9535E-07	1.9710E-08	fMe2-fMa4	
f163	1.9976E-07	1.4683E-08	fMe2-fEa2	
f164	2.1163E-07	6.5962E-07	fMe2-fVe1	
f165	2.3145E-07	4.2599E-07	fMe2-fEa1	
f166	2.4064E-07	2.8606E-08	fMe3-fVe3	
f167	2.4629E-07	1.9804E-08	fMe2-fMa1	
f168	2.5513E-07	1.3994E-08	fMe2-fJu3	
f169	2.5780E-07	3.8931E-07	fMe2-fJu2	Me2f1
f170	2.6047E-07	9.5571E-06	fMe2-fJu1	Me2f3
f171	2.6207E-07	4.2197E-07	fMe2-fSa1	Me2f4
f172	2.6314E-07	4.1113E-07	fMe2	Me2f5
f173	3.4320E-07	1.9521E-07	fMe3-fVe1	
f174	3.6302E-07	1.2565E-07	fMe3-fEa1	
f175	3.7786E-07	5.8829E-09	fMe3-fMa1	

f176	3.8937E-07	1.4500E-07	fMe3-fJu2	Me3f1
f177	3.9203E-07	2.8085E-06	fMe3-fJu1	Me3f3
f178	3.9471E-07	1.2476E-07	fMe3	Me3f5
f179	4.2372E-07	1.6211E-08	fMe4-fVe2	
f180	4.7477E-07	2.1981E-08	fMe4-fVe1	
f181	4.9459E-07	1.4104E-08	fMe4-fEa1	
f182	5.2094E-07	6.4344E-08	fMe4-fJu2	Me4f1
f183	5.2361E-07	3.2016E-07	fMe4-fJu1	Me4f3
f184	5.2520E-07	1.4996E-08	fMe4-fSa1	Me4f4
f185	5.2627E-07	8.6946E-09	fMe4	Me4f5
f186	6.0634E-07	2.3394E-09	fMe5-fVe1	
f187	6.2616E-07	1.5347E-09	fMe5-fEa1	
f188	6.5251E-07	2.0451E-08	fMe5-fJu2	Me5f1
f189	6.5518E-07	3.5215E-08	fMe5-fJu1	Me5f3
f190	6.5784E-07	1.3956E-09	fMe5	Me5f5

Table 5.1 with the spectral components of the solar parameter *aes* reveals to us a lot of "secrets" of the processes that take place in our planetary system. The first and most important is that the motion of an AB in the planetary system (including the Sun) is influenced by the motion of the ***outer AB***. For the telluric planets, the influence of Jupiter (the strongest), Saturn, followed by Uranus and Neptune is obvious.

In table 5.2 we extracted from the series of spectral components above the important ones, which we ordered by amplitude, thus having the possibility to observe and understand the weight of the planets at the solar orbital acceleration. In column 5 of the table we see the percentage share of the planetary contribution relative to the sum of all these contributions to the solar parameter *aes*. We observe the major contribution of both Jupiter and the telluric planets[12] (Mars excluded), especially of Venus.

Table 5.2

No.	Combination	Frequency [Hz]	*aes* amplitude	rel. val. [%]
1	fVe1-fJu1	4.8837E-08	1.1721E-04	11.702
2	fJu1	2.6724E-09	7.9417E-05	7.929
3	fJu1-fSa1	1.5950E-09	7.8977E-05	7.885
4	fEa1-fJu1	2.9016E-08	7.6631E-05	7.651
5	fMe1-fJu1	1.2890E-07	2.5984E-05	2.594
6	fMe2-fJu1	2.6047E-07	9.5571E-06	0.954
7	fJu1-fSa2	5.2285E-10	9.3117E-06	0.93
8	fVe1	5.1509E-08	5.2981E-06	0.529
9	fVe1-fSa1	5.0432E-08	5.2159E-06	0.521
10	fVe1-fJu2	4.6164E-08	5.1759E-06	0.517
11	fVe1-fSa4	4.7242E-08	5.1709E-06	0.516
12	fVe1-fEa1	1.9821E-08	5.1482E-06	0.514
13	fJu2	5.3447E-09	4.3231E-06	0.432
14	fVe2-fJu2	9.7673E-08	3.7820E-06	0.378

[12] When we talk about the major contribution of the telluric planets, we refer exclusively to the solar parameter *aes*.

15	fJu2-fSa1	4.2673E-09	3.6127E-06	0.361
16	fMa1-fJu1	1.4175E-08	3.4446E-06	0.344
17	fEa1-fSa1	3.0611E-08	3.4374E-06	0.343
18	fEa1-fJu2	2.6343E-08	3.4216E-06	0.342
19	fEa1	3.1688E-08	3.3787E-06	0.337
20	fEa1-fSa4	2.7421E-08	3.3538E-06	0.335
21	fSa1	1.0774E-09	3.2848E-06	0.328
22	fJu1-fUr1	2.2921E-09	2.8750E-06	0.287
23	fMe3-fJu1	3.9203E-07	2.8085E-06	0.28
24	fEa2-fJu1	6.0704E-08	2.6841E-06	0.268
25	fMe1-fVe1	8.0060E-08	1.7808E-06	0.178

6 - Formants

6.1 - Introduction

The frequency distribution of the value of a solar parameter resulting from the FFT spectral analysis is a *spectrum*. In this spectrum we observe certain forms (hence the name of *formants*), which are repeated and which represent invariant relations between certain spectral components.

Comment 6.1.1: In the Explanatory Dictionary of the Romanian Language the term *formant* is defined as an “*area of maximum relevance of an acoustic spectrum*”, but being also a spectrum, the same term can also be used in the case of the spectral components of the solar motion.

From the analysis made so far, three types of formants resulted:

- Formants of giant planets;
- Formants of the telluric planets;
- Modulation formants.

The formants of the giant planets contain invariant relations between the spectral components of the parameters of the Sun’s motion caused only by the motions of the giant planets, and the formants of the telluric planets contain invariant relations between the spectral components of the solar parameters caused by the telluric planets and the giant planets. The modulation formants contain the harmonics of a central frequency (the equivalent of the carrier to the classical amplitude modulation of a signal), components symmetrically arranged at distances equal to the frequency of the modulating signal.

6.2 - The formants of the giant planets

The formants of the giant planets are presented in the clearer version of ignoring the perturbations induced by Table 1.5.1.B in their motion, in which case the solar spectrum is much clearer. In this case the solar spectrum *aes* is given in Fig. 6.2.1 (Fig. 5.2 on which the areas belonging to formants A ... F were delimited, with the components indicated in Table 5.1). The notation of the formants for the giant planets begins with the letter G followed by the indicative of the formant (A...F) and that of the frequency. For telluric planets the name of the formant is that of the planet followed by the frequency indicator.

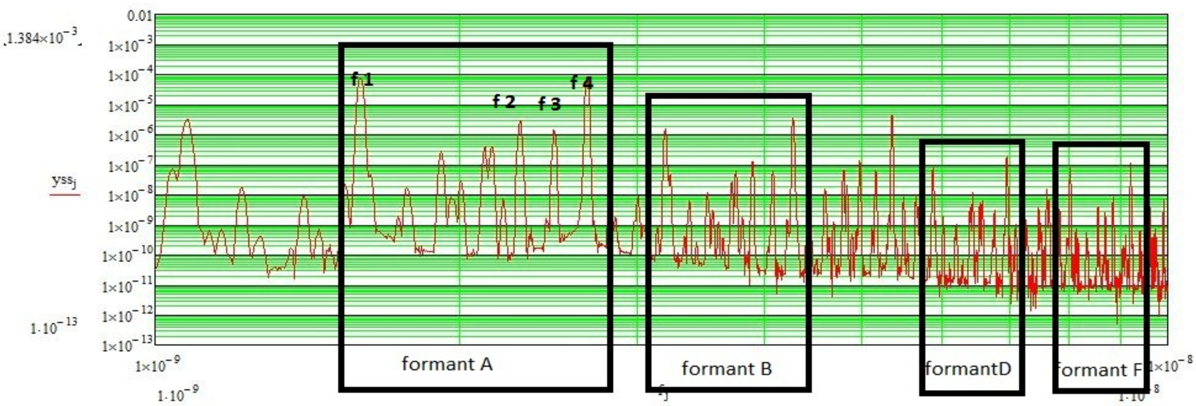

Fig. 6.2.1

Table 6.2.1 The formants of the giant planets

No.	A Formant	A Components	B Formant	B Comp.	C Formant	C Comp.
f1	1.5950e-09	fJu1-fSa1	3.1900e-09	f1+f1 fJu2-fSa2	4.2674e-09	fJu2-fSa1
f2	2.2974e-09	fJu1-fUr1	3.8924e-09	f2+f1 fJu2-fUr1-fSa1	4.9645e-09	fJu2-fUr1
f3	2.4796e-09	fJu1-fNe1	4.0746e-09	f3+f1 fJu2-fNe1-fSa1	5.1493e-09	fJu2-fNe1
f4	2.6724e-09	fJu1	4.2674e-09	f4+f1 fJu2-fSa1	5.3447e-09	fJu2
No.	D Formant	D Comp.	E Formant	E Comp.	F Formant	F Comp.
f1	5.8623e-09	fJu3-fSa2	6.9397e-09	fJu3-fSa1	8.0118e-09	fJu3
f2	6.5594e-09	fJu3-fUr1-fSa1	7.5999e-09	fJu3-fUr1	8.8357e-09	fMa1-fJu3
f3	6.7443e-09	fJu3-fNe1-fSa1	7.8217e-09	fJu3-fNe1	9.0892e-09	
f4	6.9397e-09	fJu3-fSa1	8.0118e-09	fJu3	9.1948e-09	fJu4-fUr4

We notice that the frequencies (spectral components) that enter a formant of the giant planets are four in number, and the types of formants are six in number, marked with A, B, C, D, E and F in Table 6.2.1 and Fig. 6.2.1. Table 6.2.1 indicates the four frequencies, the frequency value, and the comment columns indicate the composition of each frequency.

Comment 6.2.1: The situation of the formants is a bit more complicated because their overlaps appear. For example, formant C partially overlaps B, formant D partially overlaps E, and in formant F a component of the Martian formant appears. There is also a formant of the giant planet - the formant G - not included in Table 6.2.1 which contains components formed with fJu4 (f60, f62, f64) and which is partially overlapping with the Mars formant. As a general observation regarding the formants of the giant planets, taking into account the information regarding the formants of the telluric planets where higher order formants will also appear, we could say that in fact the formants of the giant planets are higher order formants of Jupiter.

6.3 - The formants of the telluric planets

The structure of the formants of the telluric planets is the same for all planets of this type, but as an example we chose only two clearer ones, namely those of the planets Earth and Venus, represented in Fig. 6.3.1 (an enlarged fragment from Fig. 5.3):

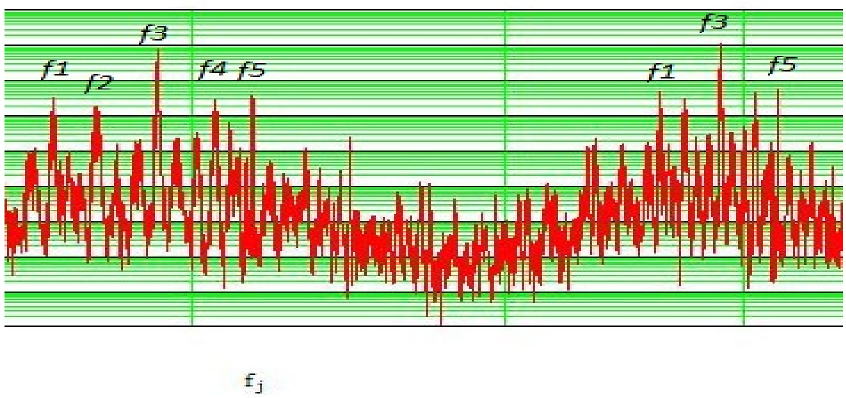

Fig. 6.3.1

Table 6.3.1 The formants of the telluric planets

No.	Ma Formant	Ma Comp.	Ea Formant	Ea Comp.	Ve Formant	Ve Comp.
f1	1.15028e-08	fMa1-fJu2	2.63434e-08	fEa1-fJu2	4.61643e-08	fVe1-fJu2
f2	1.25485e-08	fMa1-fSa4	2.73838e-08	fEa1-fSa4	4.72786e-08	fVe1-fSa4
f3	1.41751e-08	fMa1-fJu1	2.90157e-08	fEa1-fJu1	4.88366e-08	fVe1-fJu1
f4	1.57701e-08	fMa1-fSa1	3.06107e-08	fEa1-fSa1	5.04316e-08	fVe1-fSa1
f5	1.68475e-08	fMa1	3.16881e-08	fEa1	5.15090e-08	fVe1

Table 6.3.1 Continued

No.	Me Formant	Me Comp.	Amplitude
f1	1.26229e-07	fMe1-fJu2	1.1307e-06
f2	1.27339e-07	fMe1-fSa4	6.8178e-07
f3	1.28902e-07	fMe1-fJu1	2.4377e-05
f4	1.30497e-07	fMe1-fSa1	9.597e-07
f5	1.31569e-07	fMe1	1.1614e-06

In the formants of the telluric planets there is a central component f3 in Table 6.3.1, with a frequency equal to the difference between the first harmonic of the planet and the first harmonic of Jupiter, this frequency having the highest amplitude within the formant.

Comment 6.3.1: In table 6.3.1 the amplitudes of the components in the formant of Mercury are given for example, for the other planets they are omitted due to lack of space. However, the amplitude values for all formants are accessible in Table 5.1, where the components of the formants are indicated in the last column. In the case of the first three telluric planets (Me, Ve, Ea) we observe the appearance of higher order formants, reaching that for Mercury we have even a formant of order 5. Here we must note that the frequency resolution of the spectral analysis with the sampling period 8.36 days is not enough for the planet Mercury, which would need a shorter sampling period.

6.4 - Modulation formants

A modulation formant consists of a central component and several side components arranged symmetrically with respect to the central component. In Fig. 6.4.1 there is such an example, in which the central component is f25 (fJu1-fSa1)[13] from Table 5.1, accompanied by the 5 symmetrical harmonics. This formant is in turn an element (f1a) of the formant A of the giant planets, represented in Fig. 6.2.1.

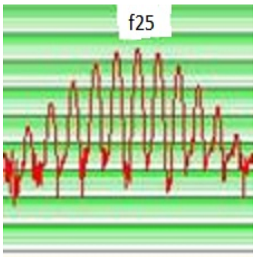

Fig. 6.4.1 f25 formant

7 - The enigma of the Table 1.5.1.B

7.1 - The individual contributions of the giant planets according to Table 1.5.1.B

In this case the mean anomaly M of the planet *i* as a function of time *t* is calculated with the equation:

$$M = L - \varpi + b \cdot t^2 + c \cdot \cos(f \cdot t) + s \cdot \sin(f \cdot t) \tag{7.1.1}$$

where *b*, *c*, *s*, *f* are given in Table 1.5.1.B reproduced below for convenience.

Table 1.5.1.B

Planet	b	c	s	f
Ju	-0.00012452	0.06064060	-0.35635438	38.35125
Sa	0.00025899	-0.13434469	0.87320147	38.35125
Ur	0.00058331	-0.97731848	0.17689245	7.67025
Ne	-0.00041348	0.68346318	-0.10162547	7.67025

[13] Modulation formant appeared when using Table 1.5.1.B which determines an amplitude modulation of each spectral component.

7.1.1 – Jupiter’s contribution to solar motion

The solar position due exclusively to Jupiter in ecliptic coordinates is given by the equations:

$$xesJu(t) = -\frac{xe(4,t)}{q_4} \cdot AU \; ; \; yesJu(t) = -\frac{ye(4,t)}{q_4} \cdot AU \quad [km] \qquad (7.1.1.1)$$

and the graphical representation in Fig. 7.1.1.1:

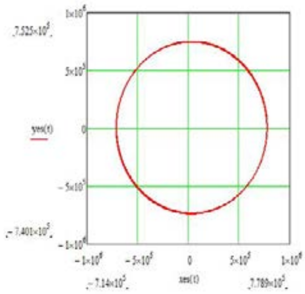

Fig. 7.1.1.1 The solar position due to Jupiter

The first-order derived distribution of the solar position (solar velocity) due to Jupiter is given by the equations:

$$vxesJu(t) = \frac{xesJu(t) - xesJu(t-\Delta t)}{Nd} \; ; \; vyesJu(t) = \frac{yesJu(t) - yesJu(t-\Delta t)}{Nd} \left[{km}/{day} \right] \qquad (7.1.1.2)$$

where *xesJu(t)* and *yesJu(t)* are given by equations 7.1.1.1, and *vxesJu(t)* and *vyesJu(t)* are the components of the solar orbital velocity due to Jupiter and represented in Fig. 7.1.1.2:

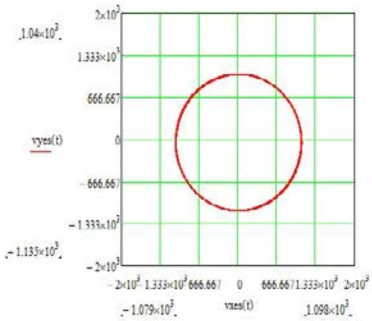

Fig. 7.1.1.2 The components of the solar orbital velocity due to Jupiter

The module of the solar orbital velocity due to Jupiter is given by the equation $vesJu(t) = \sqrt{vxesJu(t)^2 + vyesJu(t)^2}$ the representation of which is given in Fig. 7.1.1.3.

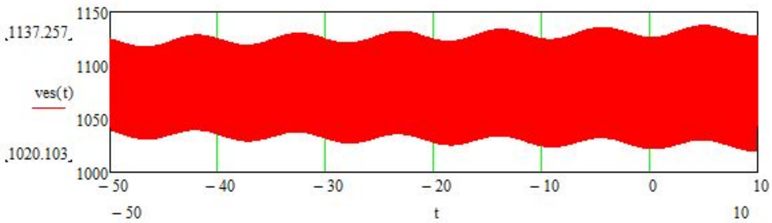

Fig. 7.1.1.3 The module of the solar orbital velocity due to Jupiter

The second-order derived distribution of the solar position due to Jupiter (solar orbital acceleration) is given by the equations:

$$axesJu(t) = \frac{vxesJu(t) - vxesJu(t-\Delta t)}{Nd} \; ; \; ayesJu(t) = \frac{vyesJu(t) - vyesJu(t-\Delta t)}{Nd} \left[{km}/{day^2} \right] \qquad (7.1.1.3)$$

where *vxesJu(t)* and *vyesJu(t)* are given by the equations 7.1.1.2, and *axesJu(t)* and *ayesJu(t)* are the components of the solar orbital acceleration due to Jupiter and represented in Fig. 7.1.1.4:

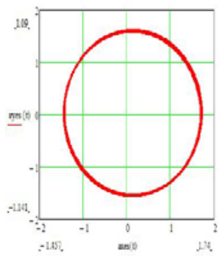

Fig. 7.1.1.4 Components of solar orbital acceleration due to Jupiter

The module of solar orbital acceleration $aesJu(t) = \sqrt{axesJu(t)^2 + ayesJu(t)^2}$ produced by Jupiter is shown in Fig. 7.1.1.5:

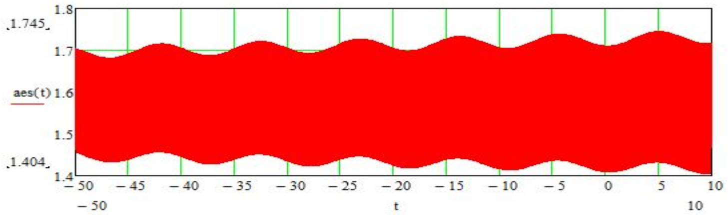

Fig. 7.1.1.5 The module of the solar orbital acceleration due to Jupiter

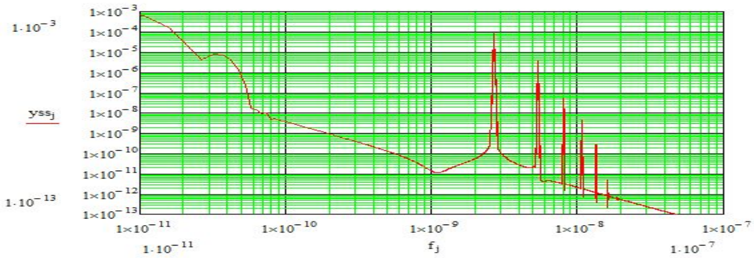

Fig. 7.1.1.6 aesJu pg spectrum

The most important spectral components in terms of amplitude of the Jovian contribution to the solar *aes* resulting from Fig. 7.1.1.6 are given in Table 7.1.1.1.

Table 7.1.1.1

Frequency [Hz]	Amplitude	Comments
fm=3.433e-011 923 years		
f1=2.6724e-009	7.7902e-005	fJu1
f2=5.3447e-009	3.8539e-006	fJu2=2fJu1 q_a=18.51
f3=8.0171e-009	5.5423e-008	fJu3=3fJu1
f4=1.0721e-008	4.3625e-009	fJu4=4fJu1
f5=1.3325e-008	2.694e-010	fJu5=5fJu1

Compared to the results of the analysis of Jupiter's contribution presented in par. 3.1 with the neglect of Table 1.5.1.B, we observe that in the case of using equation 7.1.1 there are notable differences only in the case of time distributions of solar orbital velocity and acceleration. We find that for both parameters there was a sinusoidal variation overlapping the value in the graphs in par. 3.1. This variation, following the spectral analysis, appears to us with the frequency fm in Table 7.1.1.1, to which a period of 923 years corresponds. Also the spectral components f1 ... f5 appear slightly modulated in amplitude[14].

7.1.2 – Saturn's contribution to solar motion

As we have seen in the case of Jupiter, and in the case of Saturn if we use the data in Table 1.5.1.B there are notable differences only between the graphs developed over time of the parameters of the solar orbital. In Fig. 7.1.2.1 we have the graph of the module of the solar orbital velocity due to Saturn, and in Fig. 7.1.2.2 the graph of the solar acceleration module.

[14] The components of amplitude modulation appear to us if we make an extreme zoom of fig. 7.1.1.6.

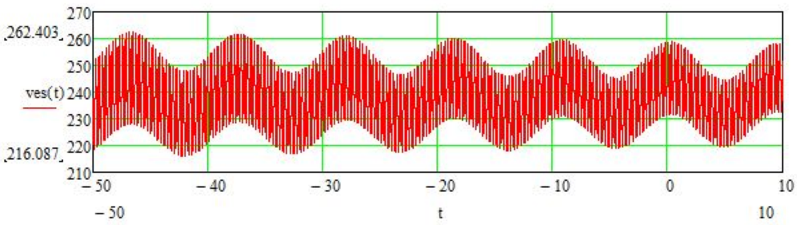

Fig. 7.1.2.1 The module of the solar orbital velocity due to Saturn

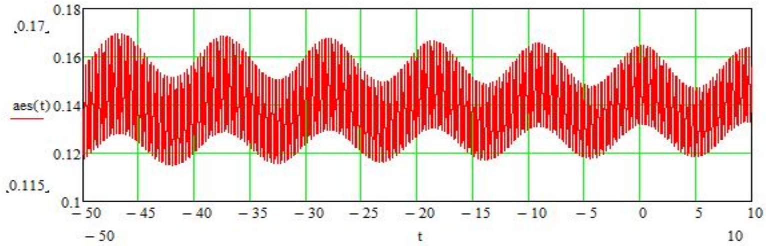

Fig. 7.1.2.2 The module of solar orbital acceleration due to Saturn

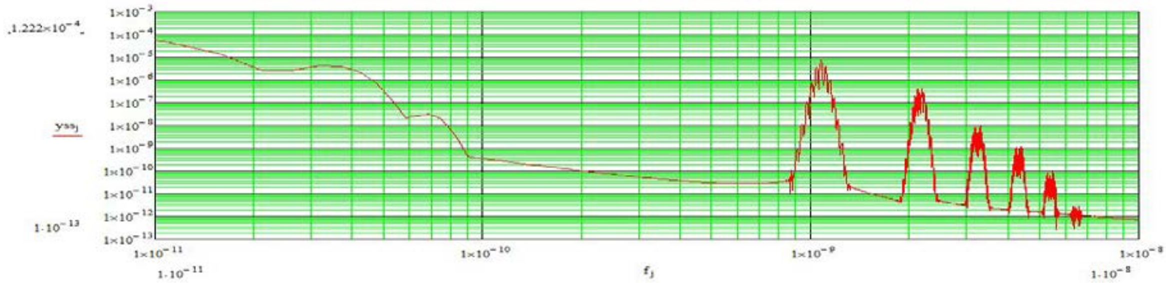

Fig. 7.1.2.3 aesSa spectrum

Table 7.1.2.1

Frequency [Hz]	Amplitude	Comments
fm1=3.433e-011 923 years		
fm2=6.8658e-011		2fm1
f1=1.0774e-009	7.557e-006	fJu1
f2=2.1495e-009	2.7573e-007	fJu2=2fJu1 q_a=27.4
f3=3.2269e-009	2.1303e-009	fJu3=3fJu1
f4=4.3043e-009	7.4496e-010	fJu4=4fJu1
f5=5.3711e-009	6.8558e-011	fJu5=5fJu1

In the case of the *aesSa* spectrum we notice that the modulation with fm is stronger than in the case of Jupiter (the second fm2 harmonic appears), and the spectral components f1 ... f5 appear to us clearly modulated in amplitude.

Comment 7.1.2.1: In the *aesSa* spectrum a modulation formant becomes visible for each spectral component, in the centre of which there is a spectral component (f1 ... f5) and the equally spaced modulation harmonics.

7.1.3 – Uranus' contribution to solar motion

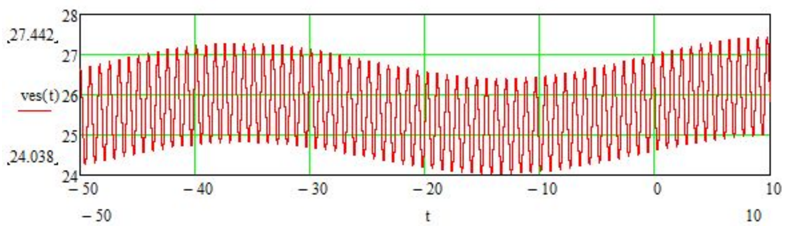

Fig. 7.1.3.1 The module of the solar orbital velocity due to Uranus

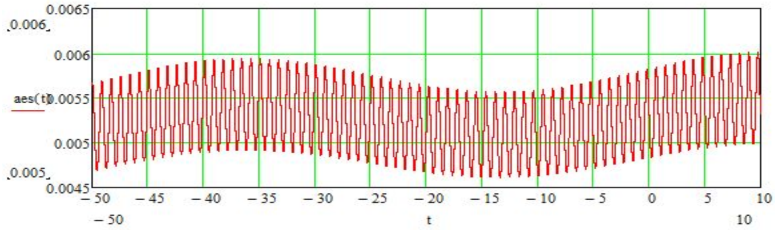

Fig. 7.1.3. 2 The module of solar orbital acceleration due to Uranus

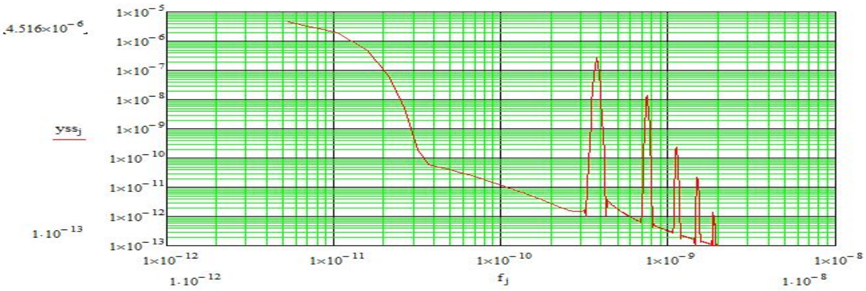

Fig. 7.1.3.3 aesUr spectrum

Due to the fact that the sampling interval of 60 centuries includes only one period of the variation of the orbital parameters of the Sun caused by Uranus, the spectral analysis is impossible. Graph 7.1.3.1 estimates a period of about 5,000 years.

Table 7.1.3.1

Frequency	Amplitude	Comments
fm3 ~1.0e-011 approx. 5000 years		estimated graphically
f1=3.7498e-010	2.6006e-007	fUr1
f2=7.4467e-010	1.4022e-008	fUr2=2fUr1 qUr=18.55
f3=1.1144e-009	2.3521e-010	fUr3=3fUr1
f4=1.4893e-009	2.2034e-011	fUr4=4fUr1

7.1.4 – Neptune's contribution to solar motion

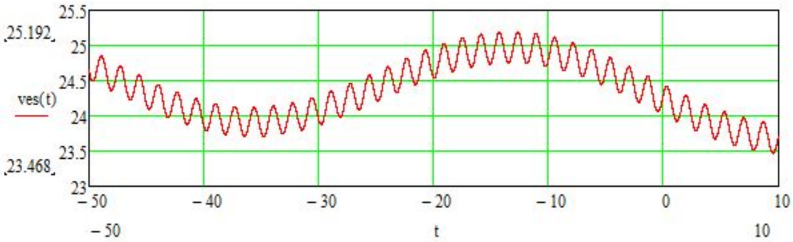

Fig. 7.1.4.1 The module of the solar orbital velocity due to Neptune

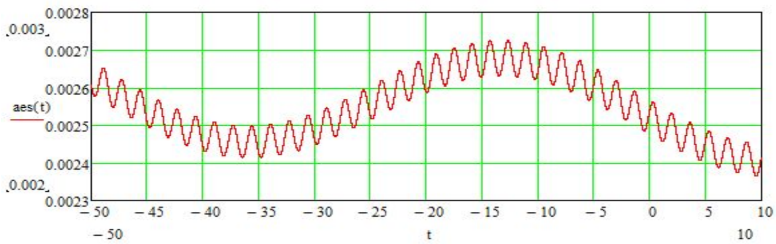

Fig. 7.1.4.2 The module of solar orbital acceleration due to Neptune

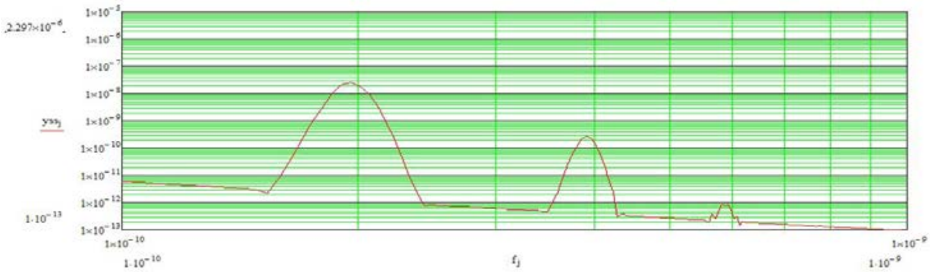

Fig. 7.1.4.3 aesNe spectrum

Due to the fact that the sampling interval of 60 centuries comprises only one period of the variation of the orbital parameters of the Sun caused by Neptune, the spectral analysis cannot highlight the modulation frequency.

Table 7.1.4.1

Frequency	Amplitude	Comments
fm3 ~1.0e-011 approx. 5000 years		estimated graphically
f1=1.9541e-010	2.5754e-008	fNe1
f2=3.9082e-010	2.6535e-010	fNe2=2Ne1 qNe=97.1
f3=5.8623e-010	7.7001e-013	fNe3=3Ne1

7.2 - The spectral components of the solar aes parameter for all planets with the corrections in Table 1.5.1.B

The solar *aes* spectrum in the variant of using Table 1.5.1.B is presented in Fig. 7.2.1 from which we can observe great differences from the one in Fig. 5.1 (reproduced in Fig. 7.2.3), especially the appearance of numerous modulation formants around all important spectral components, formants resulting from the modulation with fm1.

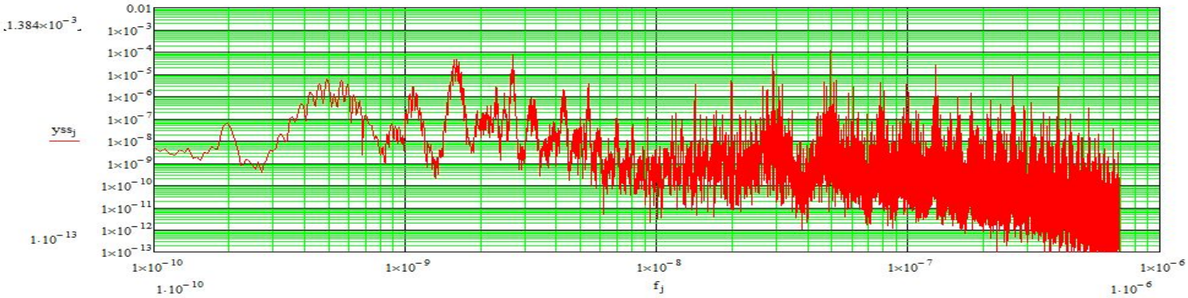

Fig. 7.2.1 aes spectrum for all planets based on the data in Table 1.5.1.B

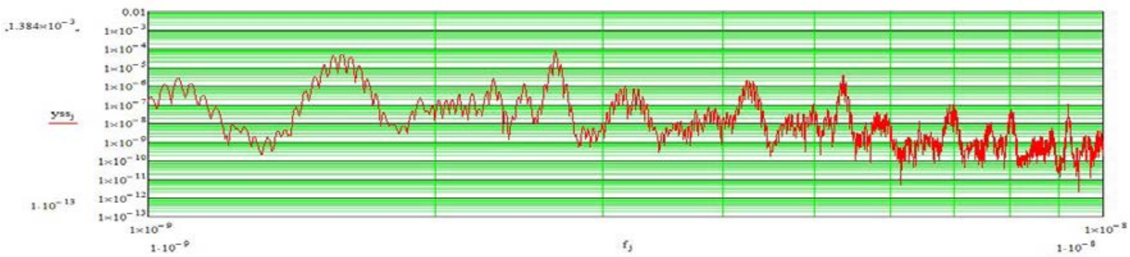

Fig. 7.2.2 aes spectrum for all planets based on the data in Table 1.5.1.B zoom1

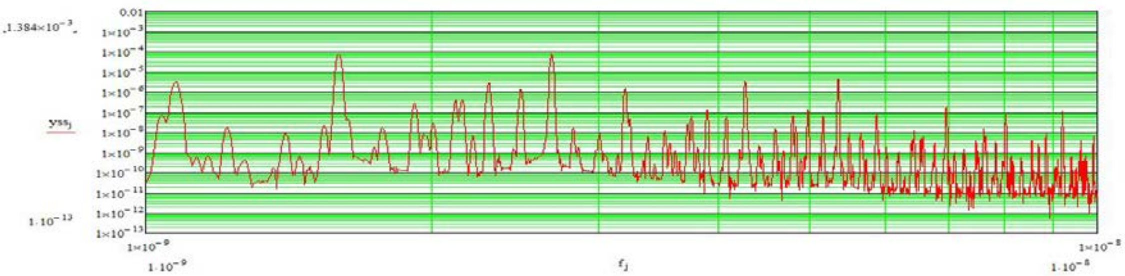

Fig. 7.2.3 aes spectrum for all planets without the data in Table 1.5.1.B zoom1

If we compare the two spectra of Fig. 7.2.2 and 7.2.3 we will understand why we preferred to do the solar orbital analysis first without the data from Table 1.5.1.B.

7.3 – The analysis of the table 1.5.1.B

The mean anomaly of the planet *i* as a function of time *t* according to [1] is given by equation 7.1.1 which we can write: $M(i,t) = M1(i,t) + M2(i,t)$, where $M1(i,t) = L(i,t) - \varpi(i,t)$ is the mean anomaly without the data in Table B, and $M2(i,t) = b_i \cdot t^2 + c_i \cdot \cos(f_i \cdot t) + s_i \cdot \sin(f_i \cdot t)$ is the correction of the mean anomaly due to Table B. The graphs of the two components of the mean anomaly for the giant planets are given in the following figures:

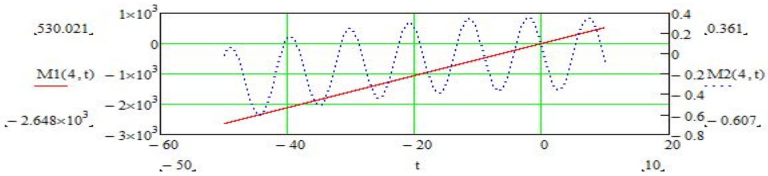

Fig. 7.3.1 Components of the mean anomaly for Jupiter

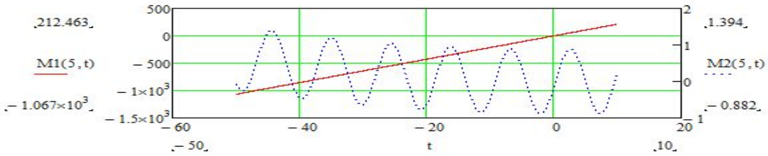

Fig. 7.3.2 Components of the mean anomaly for Saturn

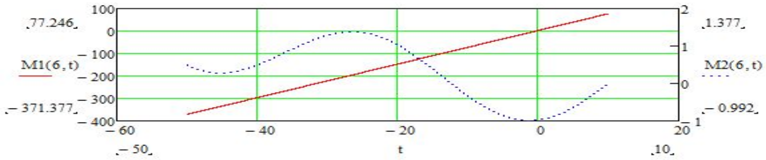

Fig. 7.3.3 Components of the mean anomaly for Uranus

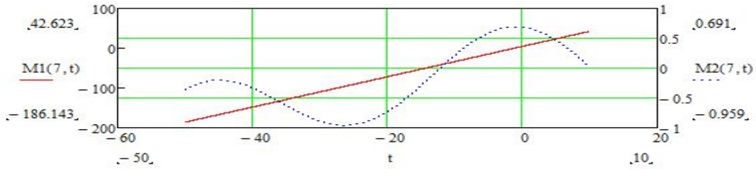

Fig. 7.3.4 Components of the mean abnormality for Neptune

The weights of the mean anomaly corrections for the giant planets resulting from Fig. 7.3.1-7.3.4 are 0.03% for Jupiter, 0.18% for Saturn, 0.53% for Uranus and 0.72% for Neptune.

Table 1.5.1.B therefore implements a periodic variation in the mean anomaly of the giant planets, a variation of the type $\alpha_i(t) + \beta_i(t)$ where:

- $\alpha Ju(t) = 0.0606406 \cdot \cos(\omega 1 \cdot t) - 0.35635438 \cdot \sin(\omega 1 \cdot t)$ for Jupiter;
- $\alpha Sa(t) = -0.13434469 \cdot \cos(\omega 1 \cdot t) + 0.87320147 \cdot \sin(\omega 1 \cdot t)$ for Saturn;
- $\alpha Ur(t) = -0.97731848 \cdot \cos(\omega 2 \cdot t) + 0.17689245 \cdot \sin(\omega 2 \cdot t)$ for Uranus;
- $\alpha Ne(t) = 0.68346318 \cdot \cos(\omega 2 \cdot t) - 0.10162547 \cdot \sin(\omega 2 \cdot t)$ for Neptune, where:
  - ω1=38.35125 degrees/century; T1=938.7 years; fm1=3.37574e-011 Hz
  - ω2=7.67025 degrees/century; T2=4693.5 years; fm2=6.75148e-012 Hz
- $\beta Ju(t) = -0.00012452 \cdot t^2$;
- $\beta Sa(t) = 0.00025899 \cdot t^2$;
- $\beta Ur(t) = 0.00058331 \cdot t^2$;
- $\beta Ne(t) = -0.00041348 \cdot t^2$

We note that ω1=5ω2, T2=5T1. The result of the Kepler's equation $\frac{a^3}{T^2} = G\frac{ms+mp}{4\cdot\pi^2} \approx 2.971\times10^{-19}$ is *a1=95.88* AU compared to 9.54 AU which is the Saturn's orbit, and *a2=280.35* AU compared to 30 AU which is the Neptune's orbit. Also, fm1=33.75e-012 Hz would correspond to the orbital with *n=5* (2^5=32), and fm2 to the orbital with *n=3* (see [4]).

Comment 7.3.1: In 2003, astronomers at the California Institute of Technology discovered the UB313 object, later called Eris[15], estimated with a diameter of 3000 km, located at about 67 AU from the Sun and with a period of about 560 years. The objects that could produce the disturbances indicated above, judged by the amplitude of these disturbances on some giant planets can only be comparable in size with them.

The components of the disturbance α for 60 centuries are given in Fig. 7.3.5, where some very interesting features are observed:

1. The time dependence of the disturbances has an elliptical shape (eccentricity ~0.99);
2. The disturbances of the planets Jupiter and Saturn are similar, but those of Saturn are larger, as are those of Neptune and Uranus, but those of Neptune are larger;
3. The ellipses describing the Ju&Sa disturbances have axes perpendicular to the axes of the Ur&Ne ellipses, i.e. they are independent of each other (this aspect together with the exact ratio of 1 to 5 of the frequencies give rise to serious suspicions regarding the artificial character of the data in Table 1.5.1.B);
4. The size of the ellipses can be related to the distance from the source of the disturbance, in other words Jupiter is farther from the source of the disturbance than Saturn, just as Uranus is farther than Neptune;

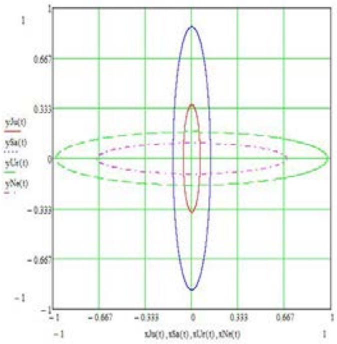

Fig. 7.3.5 Components of the disturbance α(t)

The combined influences of disturbances α and β are presented in Fig. 7.3.6:

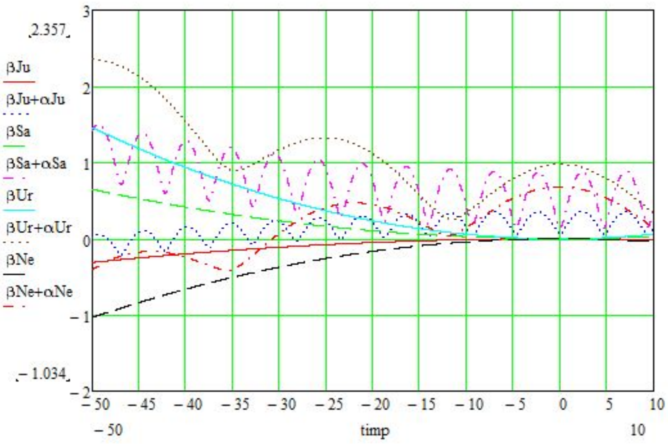

Fig. 7.3.6 Components α(t) and β(t) for 60 centuries

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[15] Patricia Daniels - The New Solar System Ice Worlds, Moons, and Planets Redefined NATIONAL GEOGRAPHIC

All these findings inevitably lead us to some conclusions:

1. Either table 1.5.1.B is an extremely elaborate farce meant to lead to fanciful assumptions the potential researcher using the data in the paper [1], in which case we should not take it into account (Indeed, if we do not take into account Table 1.5.1.B in the orbits of the giant planets no modulations appear, as we saw in par. 3);
2. Either the corrections introduced by Table 1.5.1.B are correct, i.e. they help to establish more precisely the positions of the giant planets, in which case we must take into account the causes of such disturbances, i.e. the existence of two unknown planets that populates the orbitals with *n=5* and *n=3* (see [4]).
3. In the case of the veracity of Table 1.5.1.B and the above conclusion, the silence in the media of the Jet Populsion Laboratory is somewhat understandable, because the clear proof of the existence of new giant planets in our solar system is a major discovery which must be proved by concrete astronomical observations. However, the major impediment in terms of verifying the existence of these planets is the distance they are at: 95 AU and 279 AU respectively from Earth.
4. As mentioned in comment 7.3.1, the amplitudes of the disturbances caused on the motion of some giant planets cannot be conceived as coming from some planetoids like Pluto or Eris, but also from planets comparable in size (maybe even bigger).

8 - Analysis of the influence of solar acceleration on solar activity

8.1 - Introduction

The generic term *solar activity* currently includes many variables, some periodic (number of sunspots, solar constant, etc.), others aperiodic (plasma eruptions, variations of the solar wind, etc.). This paragraph refers to periodic events, especially the number of sunspots the periodic variation of which is called the *solar cycle*.

Sunspots are depressions of the solar atmosphere (according to [8], the Wilson effect) with a temperature about 1500 K lower than the Sun's temperature (which is why they appear to us black), with the central area called the *umbra* surrounded by a less dark area called the *penumbra*. The penumbra has a structure made of shiny fibres (fibriles) about 350-700 km wide and 1500-2800 km long directed obliquely towards the umbra. All sunspots have a magnetic field from 1800 to 4000 gauss depending on their size. The typical size of sunspots is about 10,000 km and lasts from a few days to a few months.

Comment 8.1.1: Examining some physical phenomena that occur in planetary atmospheres or on the surface of the Sun and starting from the general observation that similar physical phenomena have similar causes[16], we can see a similarity between the structure of a sunspot and the structure of cyclones in the Earth's atmosphere, but especially in the atmosphere of Jupiter where they are ubiquitous. A cyclone shows a central area (the eye of the cyclone) without an apparent circulation, surrounded by an area with strong spiral circulation (the cyclone itself). The direction of rotation of the cyclone in the northern hemisphere is inverse to that in the southern hemisphere, the cyclonic movement occurring in areas with uneven latitude distribution of the speed of rotation of the atmosphere. As it is known, the Sun has an uneven latitude distribution of the axial rotation velocity, there being also the possibility of cyclones, so the objectual philosophy states that sunspots are cyclones on the surface of the Sun, the umbra being equivalent to the eye of the cyclone, and the penumbra being the equivalent of the area with spiral circulation of the cyclone.

As we saw in [6], the acceleration of a material system (MS) is the effect of a kinetic energy input following the action of an energy flux on that MS. Also in [6] we saw that force is an energy flux transmitted through the real separation surface (RSS) of a MS, a flow distributed to the inner environment of this MS. In the case of the Sun, the force is the

[16] **Maxwell J.C.** - *Matter and Motion* (1925) Ch.1.19 The general apothegm of physics: There is a maxim which is often quoted, that "The same causes will always produce the same effects." or "That like causes produce like effects."

gravitational force, produced by all the planets, a force[17] which sets the Sun in motion in its orbital.

If we talk about the motion of the Sun, we have seen so far that two types of solar acceleration have appeared:

1. Orbital acceleration as a variation of the orbital velocity of the Sun, acceleration having the direction of the tangent to the solar trajectory;
2. Radial acceleration as a variation of the radial velocity of the Sun, acceleration having the direction of the radius vector (of the solar position vector) with respect to CM.

The two types of acceleration are mutually perpendicular, we could say that they are independent of each other. The fact that the orbital acceleration manifests along the solar trajectory, and the product between this acceleration and the solar mass gives us the force acting on the Sun, led to the conclusion that the motion on a closed trajectory can be related to a rotation of a vector field (Green theorem) which we will analyse in par. 8.2.1.

Since the records of solar activity (in the form of the number of observed sunspots) have existed for only a few centuries, we have reduced the time interval of analysis from 60 centuries (validity period of the ephemerides table) to the 1750-2030 interval, an interval comprising the 25 solar cycles in the records of astronomers, cycles the list of which is given in Table 8.2.2 (columns 1-3).

8.2 – The analysis of solar orbital acceleration

In Fig. 8.2.1 the solar *aes* parameter caused by all the planets[18] is represented, in Fig. 8.2.2 the same parameter, but caused only by the giant planets, and in Fig. 8.2.3 the parameter *aes 0-7* from Fig. 8.2.1 filtered with the **medsmooth** function (*aes_0-7,183*), from which we can see that the telluric planets have a rather small contribution to the solar orbital acceleration[19].

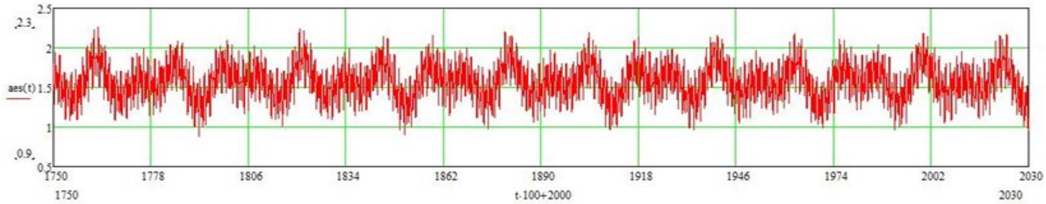

Fig. 8.2.1 aes 0-7

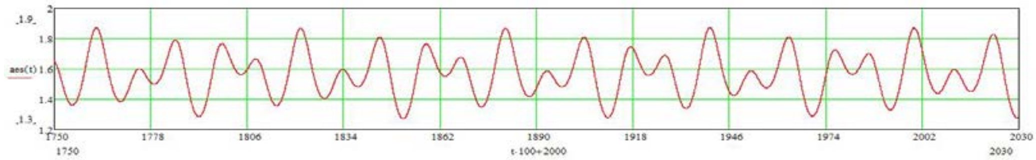

Fig. 8.2.2 aes 4-7

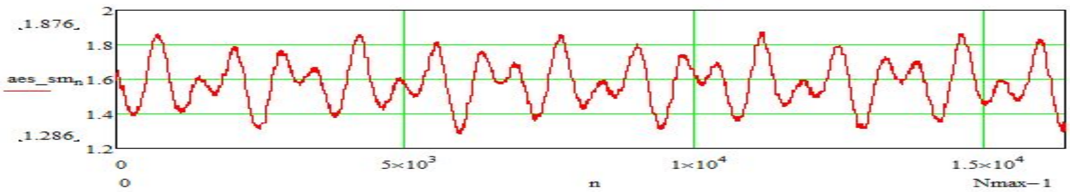

Fig. 8.2.3 aes 0-7_sm

The spectral analysis of the parameter *aes 4-7* represented in Fig. 8.2.2 has the following spectrum as a result:

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[17] The force that moves the Sun around the CM is the unique resultant of all the gravitational forces of the planets that surround it.

[18] A much expanded fragment of the graph of Fig. 2.3.2

[19] The noise shown in Fig. 8.2.1 is precisely the contribution of the telluric planets to the solar *aes*.

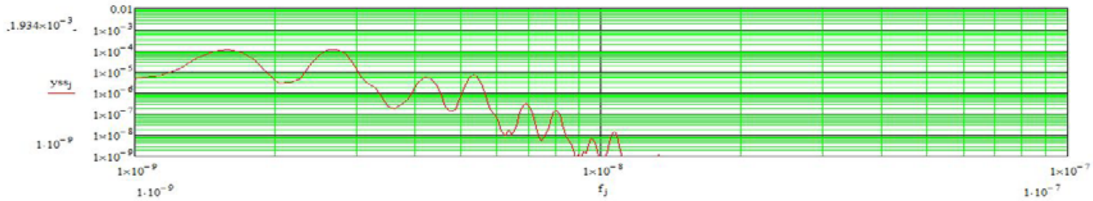

Fig. 8.2.4 aes 4-7 pg spectrum

The components of the spectrum of Fig. 8.2.4 are given in Table 8.2.1, where in the last column includes the percentage error between the exact value of the component frequency (col. 5) and the value in col. 2.

Table 8.2.1

No.	Frequency [Hz]	Period [years]	Amplitude	Components	Error [%]
f01	1.5844e-09	19.999	1.6459e-04	fJu1-fSa1 (1.5950e-09)	0.665
f02	2.6595e-09	11.915	1.6335e-04	fJu1 (2.6724e-09)	0.483
f03	4.1874e-09	7.567	7.6567e-06	fJu2-fSa1 (4.2673e-09)	1.872
f04	5.3191e-09	5.957	1.0448e-05	fJu2 (5.3447e-09)	0.479
f05	6.9035e-09	4.59 *[20]	4.3627e-07	fJu3-fSa1 (6.9344e-09)	0.446
f06	8.0352e-09	3.94	1.9511e-07	fJu3 (8.0118e-09)	-0.292
f07	9.5064e-09	3.33	9.9133e-09	fJu4-fSa1 (9.6066e-09)	1.043
f08	1.0638e-08	2.98	2.0764e-08	fJu4 (1.0684e-08)	0.431

From Table 8.2.1 we notice that the solar *aes* parameter is the “work” of only two giant planets - Jupiter and Saturn - the other giant planets Uranus and Neptune having (in the analysed time period) no contribution. Also, the amplitude of the first two components (f01, f02 with the periods T01=19,999 years and T02=11,915 years) is over 15 times higher than the amplitude of the next two (f03, f04 with the periods T03=7,567 years, T04=5,957 years) .

If instead of the exact solar acceleration *aes* we calculate the mean acceleration *ames* with the equation:

$$ames(t) = \frac{ves(t) - ves(t - \Delta t)}{Nd} \qquad (8.1)$$

where *ves(t)* is the module of the solar orbital velocity, we will obtain the *aes*-*ames* comparative graph from Fig. 8.2.5.

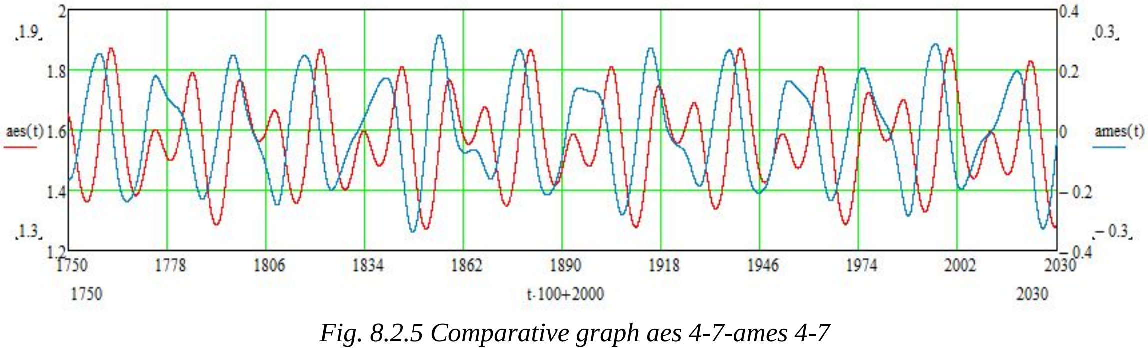

Fig. 8.2.5 Comparative graph aes 4-7-ames 4-7

From graph 8.2.5 we notice that some details from the *aes* variation fade in the case of *ames*, but the maximum and minimum years of *ames* are closer (smaller gap) to the years of the cycles in their list (columns 2 and 3).

List of years with *ames* minimums - 1766.5, 1787.9, 1809.2, 1824.6, 1847.5, 1869.5, 1885.5, 1906.9, 1928.7, 1945.4, 1965.9, 1987.9, 2002.9, 2026.1.

[20] The amplitude of the components f05 ... f08 is over 370 times smaller than f01, so it doesn't matter anymore.

List of years with *ames* maximums - 1758.8, 1775, 1796.7, 1816.7, 1839.8, 1855.1, 1877.8, 1894.6, 1914.9, 1937.2, 1954.1, 1974.9, 1995.6, 2018.6.

Table 8.2.2

Cycle no.	Minimum year list	Maximum year list	Minimum year aes 4-7	Maximum year aes 4-7	Max aes gap years	Min. year ames 4-7	Max. year ames 4-7
1	1755/02	1761/05	1755.3	1762.2	0.7		1758.8
2	1766/06	1771/05	1769.3	1774.7	3.2	1766.5	1775
3	1775/06	1778/01	1778.9	1785.2	7.1		
4	1784/09	1787/12	1792.1	1798.7	10.7	1787.9	1796.7
5	1798/04	1805/11	1804.2	1808.6	2.7	1809.2	1816.7
6	1810/08	1817/03	1814.6	1821.5	4.2		
7	1823/04	1830/04	1828.4	1833.7	3.3	1824.6	
8	1833/11	1837/01	1838.4	1844.5	7.4		1839.8
9	1843/07	1849/01	1851.3	1857.9	8.8	1847.5	1855.1
10	1855/12	1860/07	1863.5	1868.1	7.4		
11	1867/04	1870/05	1874.1	1881	10.5	1869.5	1877.8
12	1878/12	1884/01	1887.8	1892.9	8.8	1885.5	
13	1890/01	1893/08	1897.4	1903.8	10		1894.6
14	1901/12	1906/07	1910.9	1917.3	10.6	1906.9	
15	1913/06	1917/08	1922.6	1927.2	9.4		1914.9
16	1923/09	1929/12	1933.4	1940.3	10.3	1928.7	
17	1933/10	1937/02	1947.2	1952.3	15.1		1937.2
18	1944/02	1947/07	1956.8	1963.2	15.5	1945.4	1954.1
19	1954/04	1957/10	1970.1	1976.7	18.9		
20	1964/10	1968/05	1981.7	1986.5	18	1965.9	1974.9
21	1976/03	1979/01	1992.6	1999.6	20.5		
22	1986/07	1990/08	2006.4	2011.3	20.5	1987.9	1995.6
23	1996/08	2000/07	2016.1	2022.6	22	2002.9	
24	2008/11		2029.5		20.5		2018.6

Examining Figures 8.2.1 ... 8.2.3 and the data in Table 8.2.2 we can see a remarkable fact: the number of minimum and maximum values in the graphs of solar orbital acceleration is identical to the number of cycles of solar activity in Table 8.2.2. However, the dates (years) at which the respective events take place present an exaggeratedly large progressive gap (about 20 years in the last cycles), which cannot be explained yet.

Comment 8.2.1.1: If we look at Fig. 8.2.1 we see that the *aes* amplitude due to the giant planets is preponderant and the amplitude of the telluric component (noise) is smaller, but without being negligible. Finally, between the relationships that model the *aes_4-7*, *ames_4-7* and *aes_0-3* values there is the relationship that models the cycles of sunspots closer to reality, a relationship that could allow the prediction of this phenomenon.

8.2.1 The analysis of the influence of solar orbital acceleration on solar activity using Green's theorem

Two theorems are known in the calculation of vector fields: Green and Stokes, both defining a relationship between the circulation of a vector on a closed curve and the rotor (curl) of that vector on the surface delimited by the respective curve. In the case of the Green theorem the surface is flat, and in the case of Stokes the theorem is valid for any surface bounded by the closed curve. Because the solar motion[21] takes place along a flat curve that includes a flat surface, it is enough to use the Green theorem. Let a vector be $\overline{F}(x,y) = P(x)\cdot\overline{i} + Q(y)\cdot\overline{j}$, ($\overline{i},\overline{j},\overline{k}$ being the unit vectors of the X,Y,Z axes) moving on a closed curve *C*, a curve bordering a flat surface *R*.

Green's theorem tells us that:

[21] It is about the solar motion due to a certain planet.

$$\oint_C (Pdx + Qdy) = \oint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA \qquad (8.2.1.1)$$

or:

$$\oint_C \overline{F} \cdot d\overline{l} = \oint_R (\nabla \times \overline{F}) \cdot \overline{k}\, dA \qquad (8.2.1.2)$$

where $d\overline{l} = dx \cdot \overline{i} + dy \cdot \overline{j}$ and $\nabla \times \overline{F} = curl\overline{F}$ is the rotor of vector $\overline{F}$, *dA* being the area element of the surface *R*.

In our case the vector $\overline{F_i} = m_S \cdot \overline{a}_{esi}$ where m_S it is the mass of the Sun, and $\overline{a}_{esi}$ is the solar orbital acceleration in ecliptic coordinates caused by planet *i*, in other words $\overline{F_i}$ is the gravitational force exerted by planet *i* on the Sun. In the left member of equation 8.2.1.2 the product $\overline{F} \cdot d\overline{l}$ represents the elementary mechanical work of gravitational force, and the integral on the *C* curve of this mechanical work means the energy transferred to the Sun over a period of time by planet *i*. This energy has a rotational component, a component that can influence both the orbital velocity of the Sun and its internal rotational motion (see par. 8.1).

Let us return to the purpose of using the Green's theorem, namely - the genesis of solar activity following the motions of the Sun caused by the motions of the planets around it. This is because the solar motion takes place along the plane elliptical trajectories, one for each planet, ellipses of known shape and size, the surface delimited by each ellipse *i* having the rotor (curl) of the force of attraction between the planets p_i and the Sun distributed on it. The presence of a rotor (curl) field inside each solar trajectory is the reason for a possible rotational influence on the internal solar motions, motions that can stimulate the Sun's own rotational motion (or part of it) and hence the number of cyclones on the solar surface (number of spots). Let's make some numerical estimates starting from the following data:

Sun's radius $R_S = 6.9599 \cdot 10^5$ km to which a solar equatorial area $S_S = 1.5218 \cdot 10^{12}$ km^2 corresponds. The area of the solar orbit caused by the planet *i* is $S_i = \pi a_i b_i$, and the ratio between the area of the solar orbit and the solar area is $k_i = S_i / S_S$.

Table 8.2.1.1

i	Planet	a_i [km]	b_i [km]	S_i [km^2]	k_i	$aesm_i$ [m/s^2]	$Pmed_i$ [J/s ; W]
1	Mercury	9.614	9.408	284.152	$3.94*10^{-11}$	$7.395*10^{-9}$	$1.155*10^{20}$
2	Venus	264.874	264.868	$2.204*10^{5}$	$1.448*10^{-7}$	$2.774*10^{-8}$	$4.730*10^{21}$
3	Earth	456.028	455.476	$6.525*10^{5}$	$4.288*10^{-7}$	$1.804*10^{-8}$	$3.256*10^{21}$
4	Mars	73.561	73.240	16925.67	$1.112*10^{-8}$	$8.459*10^{-10}$	$1.307*10^{19}$
5	Jupiter	$7.431*10^{5}$	$7.422*10^{5}$	$1.733*10^{12}$	1.139	$2.108*10^{-7}$	$5.227*10^{24}$
6	Saturn	$2.225*10^{5}$	$2.221*10^{5}$	$1.552*10^{11}$	0.102	$1.929*10^{-8}$	$5.622*10^{22}$
7	Uranus	$1.255*10^{5}$	$1.254*10^{5}$	$4.944*10^{10}$	0.0325	$7.098*10^{-10}$	$4.196*10^{20}$
8	Neptune	$2.329*10^{5}$	$2.329*10^{5}$	$1.704*10^{11}$	0.112	$3.399*10^{-10}$	$1.904*10^{20}$

Column 7 of Table 8.2.1.1 contains the values of the mean solar ecliptic acceleration expressed in m/s^2 produced by planet *i*, and in column 8 we find the intensity of the energy flux transmitted to the Sun by planet *i* (an intensity equivalent to the mean power Pmed).

Comment 8.2.1.1: If we know the mean solar ecliptic acceleration caused by the planet *i* $aesm_i$ then we can calculate the mean force $Fmed_i = m_S * aesm_i$, (where m_S is the mass of the Sun), and the product between this force and the length of the solar elliptical trajectory due to the planet *i* (L_i) gives us the mechanical work $W_i = Fmed_i * L_i$, i.e. the energy transferred to the Sun during a period by that planet. The ratio between that energy and the length of the period *Ti* gives us the mean power $Pmed_i$.

Solar ecliptic acceleration due to Jupiter *aesJu* = 1.4223... 1.7251 km/day^2 gives us an average value of $2.108*10^{-7}$ m/s^2, 11 times higher than that produced by Saturn. The fact that the area of the ellipse bordered by the solar trajectory produced by Jupiter is larger than the equatorial solar area means that the rotor (curl) of the Sun-Jupiter attraction force is distributed over an area that includes both the solar interior and the solar atmosphere (it lays for another $4.666*10^4$ km outside the Sun).

The area of the solar orbit due to Neptune is $S_7=1.7041*10^{11}$ km^2 (11.2% from the solar area), but the solar orbital acceleration produced is very small (*aesNe* = 0.00249... 0.00258 km/day^2), 60 times smaller than that produced by Saturn.

The result of these estimates proves that the possible influence of the planets on solar activity is largely given by the planets Jupiter and Saturn, but mainly by Jupiter.

8.3 – The analysis of solar radial acceleration

For the analysis of the solar radial acceleration, we will restrict the analysis interval of the solar distance distributions from the CM to the period 1590-2050, thus including both the period of known solar cycles and the interval known as “Maunder minimum” (1645 - 1715) in which an absence (or a small number) of sunspots is mentioned. In Fig. 8.3.1 we have the distribution of the solar distance relative to CM, in Fig. 8.3.2 the radial velocity distribution and in Fig. 8.3.3 the solar radial acceleration.

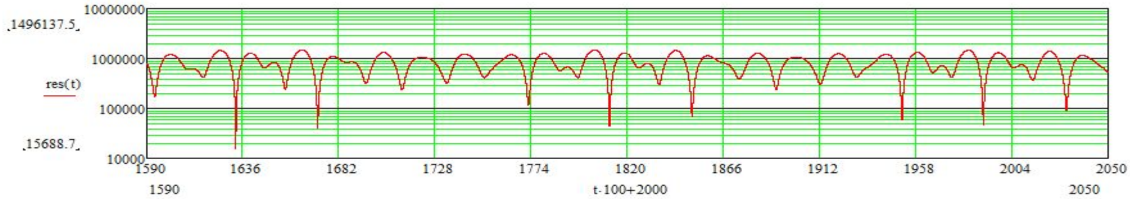

Fig. 8.3.1 res(t) 1590-2050 log

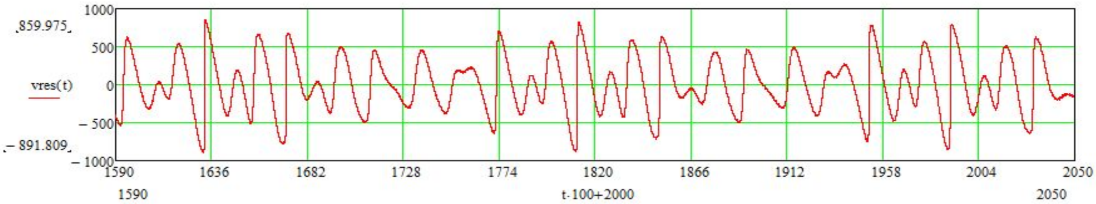

Fig. 8.3.2 vres(t) 1590-2050

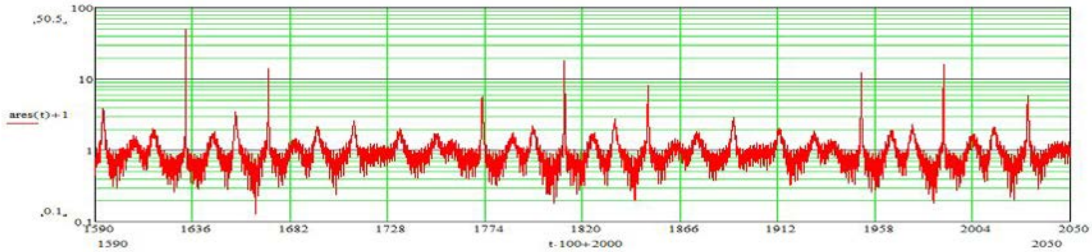

Fig. 8.3.3 ares(t) 1590-2050 log

We saw it in par. 2.4 where we analysed the distributions of the solar distance from the CM that the *ares(t)* parameter (solar radial acceleration in ecliptic coordinates, see Fig. 2.4.3) has an extremely uneven time distribution, having at certain moments very high values.

From Fig. 8.3.3 (an expansion of Fig. 2.4.3) we notice that the radial acceleration has some peaks grouped three by three (triplets), of which the central one is larger. The time distance between the central peaks is 178.8 years (period called the “José cycle” in [7]), and between the peaks of a triplet 39 years. The highest peak of *ares(t)* from the year 1632.6 with amplitude 50 could mark the beginning of the Maunder interval, based on the assumption that radial acceleration seems to inhibit periodic solar activity and not to stimulate it.

Comment 8.3.1: The mean value of the peak of the solar radial acceleration of 50 km/day^2 is only 14.923 km/day^2 if we calculate the mean acceleration as Δv/Δt, where Δv and Δt are established on the basis of the expanded graph in Fig. 8.3.2. We will convert the acceleration values from km/day to m/s: 1 km/day=0.01157 m/s, 1 km/day^2=1.33959e-07 m/s^2. To an acceleration of 14.923 km/day^2 another of 1.999e-06 m/s^2 corresponds. Given that the mass of the Sun is 1.9891e30, the result is a force (F=ma) of 3.978e24 N, i.e. an energy flow of 3.978e24 J/s provided to the solar inner environment, an aperiodic flux that cannot influence the periodic processes[22] within the so-called "solar activity", but can disrupt them. However, it is possible that certain aperiodic events of solar activity (explosions, abnormal plasma emissions, etc.) are caused by the particularly high energy input of these acceleration peaks.

9 - Conclusions

1. The motion of a AB in the planetary system (including the Sun) is influenced by the motion of ***outer ABs***. One of the most unexpected consequences of this observation is the influence of the Earth on Mercury's motion (see Table 3.8.1), in addition to the influence of the giant planets;

2. In [5] due to the fact that the modelling of planetary motions was purely theoretical (without a real time support) the importance of solar radial acceleration in periodic solar activity was much overestimated. In the present study in which the real-time motion of the planets according to the ephemerides tables is analysed, the essential influence on the periodic solar activity of the solar orbital acceleration and the much reduced influence of the radial acceleration could be found;

3. From the tables with planetary orbital frequencies some interesting details are observed:

- fSa0-fUr0 = 6.9855E-10 Hz (frequency to which a period T=16568.71 days corresponds, error 1.13% compared to 2^{14} days);
- fSa0-fNe0 = 8.834395E-10 Hz (T=13101.15 days, error -0.046% compared to $2^{17}/10$) days.

It should be noted that these are natural planetary frequencies (inverse of periods) and not spectral components, the result of which is other relationships involving powers of 2 compared to the relationships discussed in [4];

4. The analysis of the solar orbital acceleration caused by the gravitational forces of the planets surrounding the Sun showed (see Table 8.2.1.1) that the mean gravitational power received by the Sun from all the planets is $5.292*10^{24}$ J/s, of which 98.77% from Jupiter, 1.06% from Saturn, 0.09% from Venus, 0.06% from Earth and cumulated under 0,01% from the other planets, this being the proportion in which the planets in the solar system can influence the periodic solar activity.

5. The analysis of the solar radial acceleration in the interval 1590-2050 did not show any effect on the periodic solar activity, except for the possible suppression of this activity in the interval known as "Maunder minimum" (1645 - 1715) following a massive pulse of *ares* from 1632 (equivalent to an energy input of $3.978*10^{24}$ J/s, see comment 8.3.1). However, the massive energy input provided to the entire solar structure by radial acceleration pulses can influence the processes inside the Sun, an influence that manifests itself on the outside in the form of plasma eruptions or variations in the solar wind.

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[22] A periodic process can also be influenced by a periodic intervention (interaction), either constructive (synphasic) or destructive (antiphasic).