Solar Orbital and Solar Activity

Aurel Rusu Duma (<u>rusuduma@yahoo.com</u>) Translated from Romanian by Viorica Zamarcaru (<u>zamarcaruv@yahoo.com</u>)

1 Introduction

In the article *Planetary Orbitals* we have seen that each astronomic body (AB) within the structure of a planetary system (PS) has an associated orbital, notion we remind the reader: *The abstract object made-up from the reunion of all the <u>invariant</u> attributes which is characteristic for the motion of an AB within a PS, shall be called orbital. An orbital is composed of the following invariant attributes:*

- Spatial zone where the orbital motion is ranged.
- Orbital period or its reverse *orbital frequency*.

As the Sun also is an AB of PS, even its central element, the Sun also will have an orbital, i.e. a spatial area (a volume) which encloses the motion (trajectory) of the Sun around the mass center (MC) of PS. Also, this motion will be assigned a frequency distribution (a spectrum), as we will see later.

Comment 1.1: As established in *Introduction to objectual philosophy*, the mass center of a planetary system is the internal translation reference T of the system, the null rotation (revolution) point and application point of the resultant of all interaction forces with the planetary system external systems. For a closer star, or for the galactic center, the whole mass of our planetary system is concentrated in this point. *Table 1.1*

i	AB	<i>m</i> [kg]	<i>a</i> [UA]	Е	q	T [days]
-	Sun	$1.989 \cdot 10^{30}$	-	-	-	-
1	Mercury	$3.302 \cdot 10^{23}$	0.38709893	0.2056	6023600	87.969
2	Venus	$4.869 \cdot 10^{24}$	0.72333199	0.0068	408523.5	224.701
3	Earth	$6.042 \cdot 10^{24}$	1.00000011	0.0167	328900.5	365.256
4	Mars	6.419 <i>·</i> 10 ²³	1.52366231	0.0934	3098710	686.980
5	Jupiter	1.899 <i>·</i> 10 ²⁷	5.20336301	0.0484	1047.355	4332.589
6	Saturn	$5.685 \cdot 10^{26}$	9.53707032	0.0541	3498.5	10759.22
7	Uranus	$8.683 \cdot 10^{25}$	19.19126393	0.0472	22869	30685.4
8	Neptune	$1.024 \cdot 10^{26}$	30.06896348	0.0086	19314	60189
9	Pluto	$1.25 \cdot 10^{22}$	39.48168677	0.2488	1.5912*10 ⁸	90465

In Table 1.1 the main attributes of AB of our planetary system structure are given, where m is the mass of AB, a is the major semi-axis of the orbit, ε is the numerical eccentricity of the orbit, T is the orbital period and q will be defined below. If we note the sun mass as m_s and planets masses as m_i , (*i* is the index in Table 1.1 starting with 1 for Mercury¹) then $q_i = m_s/m_i$.

¹ The soft for mathematical modeling was Mathcad, and the convention for the indices of a vector was ORIGIN=1.

2 The motion of PS elements in relation to the MC of the planetary system

Given a PS represented in Fig. 2.1, consisting of small-sized *n* AB (planets), with masses m_1 , m_2 , ... m_n orbiting around a large-sized AB (Sun) with mass m_s . To an external 2D (twodimensional) reference system with O origin and XY² axes, the PS elements have the vectors of spatial position $\overline{r_1}$, $\overline{r_2}$,... $\overline{r_n}$ and $\overline{r_s}$.



Fig. 2.1

In these conditions, the mass center (MC) of the PS has the position vector \overline{r}_{MC} given by the equation³:

$$\overline{r}_{MC} = \frac{m_1 \overline{r_1} + m_2 \overline{r_2} + \ldots + m_n \overline{r_n} + m_S \overline{r_S}}{m_1 + m_2 + \ldots + m_n + m_S}$$
(2.1)

If we move (shift) the O origin of the reference system in the MC, this is equivalent to writing in the equation 2.1 $\overline{r}_{MC} = 0$. Denoting $m_1 + m_2 + ... + m_n + m_s = m_T$, where m_T is the total mass of the PS, the equation 2.1 becomes:

$$\frac{m_s}{m_T}\overline{r}_s = -\frac{m_1}{m_T}\overline{r}_1 - \frac{m_2}{m_T}\overline{r}_2 - \dots - \frac{m_n}{m_T}\overline{r}_n$$
(2.2)

hence the position vector of the Sun to the MC:

$$\overline{r}_{s} = -\frac{m_{1}}{m_{s}}\overline{r_{1}} - \frac{m_{2}}{m_{s}}\overline{r_{2}} - \dots - \frac{m_{n}}{m_{s}}\overline{r_{n}}$$

$$(2.3)$$

By denoting $\frac{m_s}{m_i} = q_i \ i \in [1, n]$, the equation 2.3 becomes:

$$\overline{r}_{s} = -\sum_{i=1}^{n} \frac{\overline{r}_{i}}{q_{i}}$$
(2.4)

We know that planets move in elliptical trajectories with MC in the focus point and $\overline{r_i}$ as the position vector, hence the Sun shall also move throughout the elliptical trajectories⁴, with

 $^{^{2}}$ We consider the simplified case where the motions of the AB forming the PS are coplanar, included in the XY plane.

³ Richard Fitzpatrick - An Introduction to Celestial Mechanics, Cambridge University Press 2012

⁴ Sun trajectory is elliptical in a couple consisting of the Sun and a certain planet, but on the whole, by the composition of all elliptical motions, solar trajectory is more complex, as we shall see.

MC focus point also, but with $\overline{r_s}$ as position vector. Mathematical modeling of these motions (trajectories) shall be done in simplifying conditions, namely:

1. Motion in elliptical trajectory in the XY plane is the result of the projection on the XY plane of a circular motion with radius a_i and constant angular velocity $\omega_i(t)$, motion set in a plane that includes the X axis and is inclined by the ϕ_i angle in relation to XY plane. In these circumstances we have a_i major semi-axis of the orbit collinear with the X axis, $b_i = a_i \cos(\phi_i)$ collinear with the Y axis (minor semi-axis of the orbit), $\varepsilon_i = \sin(\phi_i)$ and $\omega_i(t) = \frac{2\pi}{T_i} \cdot t$.

2. All elliptical trajectories are included in the XY plane and have the apsides axis (perihelion-aphelion axis) collinear with the X axis.

3. Starting point of trajectories calculation (t = 0) is the perihelion.

In these circumstances $(\overline{i}, \overline{j})$ being the unit vectors of X and respectively Y axis), the position vector of the Sun for a m_S - m_i couple will be:

$$\overline{r_i}(t) = x_i(t)\overline{i} + y_i(t)\overline{j}$$
(2.5)

where:

$$x_i(t) = \frac{a_i}{q_i} (\cos(\omega_i t) - \varepsilon_i)$$
(2.6)

$$y_i(t) = \frac{a_i}{q_i} \left[\sin(\omega_i t) \cdot \cos(\arcsin(\varepsilon_i)) \right]$$
(2.7)

Global (resultant) position vector of the Sun will be the vector sum of all individual position vectors ⁵, i.e.:

$$\overline{r}(t) = \sum_{i=1}^{9} \overline{r_i}(t)$$
(2.8)

or finally:

$$x(t) = \sum_{i=1}^{9} \left[\frac{a_i}{q_i} \left(\cos(\frac{2\pi}{T_i} t) - \varepsilon_i \right) \right] \cdot 1.4959787 \cdot 10^8$$
(2.9)

$$y(t) = \sum_{i=1}^{9} \left[\frac{a_i}{q_i} \left(\sin(\frac{2\pi}{T_i} t) \cdot \cos(\arcsin(\varepsilon_i)) \right) \right] \cdot 1.4959787 \cdot 10^8$$
(2.10)

With the data in Table 1.1, x(t) and y(t) resulting in km.

3 The individual contributions of planets to Sun motion

The most important couple in our planetary system is Sun-Jupiter, both in the intensity of the interaction⁶ and in the ratio of masses (q_5), which determines the largest contribution to the position vector of the Sun in its orbital. With the data in Table 1.1 and according to relations 2.9 and 2.10 (in which i=5 both initially and finally), assuming that we neglect the other planets, for the Sun the trajectory in Fig. 3.1 results. We note that on the X axis the position of the Sun in relation to MC is between $7.7844 \cdot 10^5$ km (at aphelion of Jupiter) and $7.0657 \cdot 10^5$ km at Jovian perihelion. If we consider that the Sun radius is $6.9599 \cdot 10^5$ km, then the MC of Sun-Jupiter couple is <u>outside</u> the Sun, at a distance between

⁵ The individual contributions of each planet in the PS.

⁶ See Figure 4.1 and Table 4.1 of *Planetary Orbitals*, reproduced for convenience in *Annex 1*.

 $7.7844 \cdot 10^5 - 6.9599 \cdot 10^5 = 8.245 \cdot 10^4$ km at Jovian aphelion and $7.0657 \cdot 10^5 - 6.9599 \cdot 10^5 = 1.0586 \cdot 10^4$ km at Jovian perihelion.



Fig. 3.1 - The trajectory of the Sun in relation to MC of Sun-Jupiter couple





In fig. 3.2 the variation of position vector module of the Sun $r(t) = \sqrt{x(t)^2 + y(t)^2}$ is displayed during a complete revolution period of Jupiter. The time axis is graded in days. Using the same procedure we will find in Table 3.1 for all couples Sun-planet r_{min} and r_{max} positions between which the position of the Sun varies according to each couple.

In the fourth column of Table 3.1 by adding we get r_{MM} [km], radius of spatial range within which the motion of the Sun is framed, i.e., the first attribute of solar orbital:

$$r_{MM} = 11.6 + 266.7 + 462.4 + 80.4 + 7.78 \cdot 10^5 + 4.3 \cdot 10^5 + 1.3 \cdot 10^5 + 2.35 \cdot 10^5 + 2459 \quad (3.1)$$

$$r_{MM} = 1.578 \cdot 10^6 \tag{3.2}$$

This radius is reached when all the planets are aligned to aphelion. It is important to note that due to the motion of the Sun around the MC of our planetary system, the distance between the surface of the Earth and of the Sun varies over time (in addition to the annual variation determined by Earth orbit eccentricity) by up to $1.578 \cdot 10^6 - 6.9599 \cdot 10^5 = 8.82 \cdot 10^5$ km i.e. 0.59% of the UA. This may influence in the same proportion the solar constant (the amount of energy received from the Sun to the Earth's crust).

Comment 3.1 Variation of the distance between Earth and Sun due to Sun's motion on its orbital is extremely slow due to the fact that giant planets have the most important contribution to this variation (noticeable contribution), and they have very long periods of revolution (in human time scale).

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i	Planet	r _{min} [km]	$r_{max}[km]$
1	Mercury	7.64	11.59
2	Venus	263.08	266.67
3	Earth	447.24	462.44
4	Mars	66.68	80.43
5	Jupiter	7.0657*10 ⁵	$7.7844*10^5$
6	Saturn	3.8571*10 ⁵	4.2977*10 ⁵
7	Uranus	1.1961*10 ⁵	1.3145*10 ⁵
8	Neptune	2.3089*10 ⁵	2.3489*10 ⁵
9	Pluto	$1.4789*10^{3}$	$2.4586*10^{3}$

4 Analysis of Sun's motions in its orbital

To start with we shall analyze the solar trajectory determined by telluric planets (Mercury, Venus, Earth + Moon and Mars) on a temporal interval of three complete Martian periods (2061 days). That means that we will have in the relation 2.8:

$$\overline{r}(t) = \sum_{i=1}^{4} \overline{r_i}(t) \tag{4.1}$$

In Fig. 4.1 we see the trajectory of Sun's center caused by revolution motions of the telluric planets with axes graded in km, revealing very steep variations of position, and in Fig. 4.2 the value of position vector module $r(t) = \sqrt{x(t)^2 + y(t)^2}$ in km, corresponding to the trajectory in Fig. 3.1, the time axis being graded in days. On the basis of the trajectory in Fig. 4.1 and of the value of the position vector in Fig. 4.2, we will calculate the velocity and acceleration corresponding to these motions.

Comment 4.1: When calculating velocity and acceleration two methods with the same final results can be used:

1. Classical methods of differential calculus, i.e. $v(t) = \frac{d}{dt}r(t)$ and $a(t) = \frac{d}{dt}v(t)$

2. Objectual method of calculation with finite differences (see chapter 2 of *Introduction to Objectual Philosophy*), i.e. $v(t) = \frac{r(t) - r(t - \Delta t)}{\Delta t}$ and $a(t) = \frac{v(t) - v(t - \Delta t)}{\Delta t}$, where Δt (one day) is

temporal increment in the relations 2.9 and 2.10 during the mathematical modeling. In this case, the

concept of derivative is replaced with the density concept of the derivative distribution. Using the same software for mathematical modeling - Mathcad - applying the classical method, the calculation takes tens times longer than the calculation with finite differences.

The graphs presented reveal an interesting observation, namely when the Sun position in relation to the MC passes through its minimum values, higher pronounced acceleration values appear. But we know that acceleration means energy input (in this case kinetic), energy that the Sun with all its internal structure receives from the planets around it (through gravitational interactions). In the case of telluric planets this energy is very low compared to the energy received from the giant planets (as we shall see below), but the scenario is repeated in these planets also, i.e. the maximum energy intake occurs also at times of the solar position vector minimum.



Fig. 4.4 - Acceleration in km/day^2 corresponding to velocity variation in Fig. 4.3

It is now time to consider solar trajectory determined by the giant planets (Jupiter, Saturn, Uranus and Neptune) over a temporal interval of 180567 days (three complete cycles of the planet Neptune). We will have in the relation 2.8:

$$\overline{r}(t) = \sum_{i=5}^{8} \overline{r_i}(t) \tag{4.2}$$

Fig. 4.5 shows how complex the sun's trajectory is due to giant planets revolutions and Fig. 4.6 the variation of position vector module in the same situation.



Fig. 4.6 –*Variation of the solar position vector due to giant planets [km]* Furthermore, in Figures 4.7 and 4.8 we have solar velocity and acceleration caused by giant planets motions, time axis still being in days.



Fig. 4.7 –Sun velocity derived from Fig. 4.6 [km / day]





The graphs presented clearly reveal that under the influence of giant planets also abnormally high peaks of solar acceleration appear (superposed over a quasi-sinusoidal variation) much higher accelerations than in the case of telluric planets. Also, these peaks coincide with the minimum values of solar position vector in relation to MC. To confirm the statements above in Fig. 4.9 and 4.10 the variations in Fig.. 4.6, 4.7 and 4.8 are shown in detail for the first 5000 days (13.7 years) in the interval of 180567 days (494 years).





Let us now analyze the above data in terms of those set in chap. 7 of *Introduction to Objectual Philosophy*. There we saw that *force* ultimately means an energy flux transmitted to the material system (MS) over which the force is applied. The transmitted energy flux is distributed on the internal structure of MS (internal state variation), then the external state variation appear (acceleration, see *Energetic Action* under the same chapter 7).

Sun's velocity of 1 km/day means 0.012 m/s, and acceleration of 1 km/day² means $1.389*10^{-7}$ m/s², so when the Sun has an acceleration of 2.77 km/day², it means $a=3.848*10^{-7}$ m/s². However, given that $m_s=1.989*10^{-30}$ kg, then $F = m_s \cdot a$, i.e. $F=7.654*10^{-23}$ N, i.e. an energy flux of $7.654*10^{-23}$ J/s. It is natural that this kinetic energy intake globally distributed throughout the internal structure of the Sun modify the internal solar kinetic processes (internal action of energy flux transmitted), processes whose externally visible effects appear

to us as "solar activity", i.e. spots, eruptions, variations of the magnetic field and solar wind, etc.. We also see in Fig. 4.8 that at certain times, the acceleration peak is higher than that of day 3000 (e.g. on day 133750 it reaches 10.8 km/day^2).

5 Spectral analysis of solar orbital

To determine the second attribute of solar orbital - orbital frequency - we will analyze in the (spectral) frequency domain the Sun's motion around MC. The most significant parameter of those analyzed so far is the acceleration⁷ as it gives us direct information on the energy received by the Sun during its evolution on the orbital. Therefore, we will analyze in the frequency domain the values array of solar acceleration similar to that in Fig. 4.8, but with the contribution of telluric planets also on an interval of $2^{17}=131072$ days (359 years). The algorithm for spectral analysis is the *Fast Fourier Transform* (**FFT** (**x**)) algorithm applying to an **x** vector with the number of elements of the form 2^n . In our case, the **x** vector is a(t) in Fig. 5.1.

We note that in regard to Fig. 4.8 a kind of noise appeared that actually represents the contribution of telluric planets, where we can appreciate the size of this contribution compared to the contribution of giant planets. Also, out of the length of 180567 days of Fig. 4.8, here only 131072 days are rendered for the reasons mentioned above concerning the requirements of FFT analysis.





Fig. 5.2 shows how complex the spectral distribution of solar acceleration is, where j is the index of spectral component value resulting after the FFT analysis, y_j being the spectral component module and f_j its associated frequency in Hz. Out of the spectral components amount only few (the most obvious) were selected, and their values accompanied by brief comments can be seen in Table 5.1. Examining the figures presented, especially the table, we see that the solar orbital spectrum is similar to the spectrum of amplitude modulated signal, in which, besides the main frequencies, sum and difference frequencies appear. Until now, due to the complexity of the spectrum only a few components were analyzed, enough to start with for the purpose of this article, namely that to show very close link between the motion of the Sun in its orbital and solar activity caused by this motion.

⁷ Frequencies of the spectral components resulting from the analysis are the same even if we analyze the velocity or position of the Sun, only the component amplitudes and shape of the spectrum being different.





Fig. 5.2 – Spectrum a(t) in Fig. 5.1 in unfiltered version

Table 5.1 and Fig. 5.2 show the contributions of the first three planets (the contribution of Mars is negligible) to the solar orbital spectrum. Note the major contribution of Venus, that naturally results if we review Fig. 4.1 and Table 4.1 of the *Planetary Orbitals* (for convenience we have reproduced these items in *Annex 1* to this article). From their

⁸ Spectral component amplitude varies depending on the method of analysis, analysis interval, or on applying or not a filter to the data, etc. Rapport of relative values is maintained, as well as major component frequencies.

examination it becomes very clear that the second couple in intensity of the gravitational interaction after couple Sun-Jupiter is Sun-Venus, hence the contribution of Venus to the solar orbital spectrum must also be important.

The first f_1 spectral component with periods of 19.9 years is very close to the difference between the orbital frequencies of Jupiter and Saturn. The second f_2 component with the period of 13.8 years is very close to the difference between the orbital frequencies of Jupiter and Uranus. Note that the most intense f_3 spectral component with a period of 12.8 years corresponds to the average frequency of quasi-sinusoidal variation of Sun's acceleration, frequency very close to the difference between the orbital frequencies of Jupiter and Neptune. For other components the relationship with orbital frequencies has not yet being found.

6 Possibilities for prediction of solar activity

It is time to draw the reader's attention that so far we have only made a theoretical (pure mathematical) analysis of the motion of planets with given masses and periods of revolution on elliptical trajectories, but all these motions were not real planetary motions in real time but in simplified conditions 1, 2 and 3 of par. 1. In reality, the planets move in elliptical trajectories in relation to MC of the PS, but the orbits are not coplanar and apsides axes do not coincide either. However, at present the planets trajectories are well known (without this information space missions would not be possible), so the relations 2.9 and 2.10 can be adjusted so that they be valid in real time. Annex 2 shows the apsides axes rotation simulation results to demonstrate that this rotation, in contrast with the simplifying conditions in par. 1, does not alter initial results in terms of quality. Only the temporal distribution of the acceleration peak amplitude changes but not its spectral components. Neither the relatively low inclination of planetary orbits compared to the situation when they were coplanar can change the frequency distribution of solar acceleration.

If we accurately determine the position of each planet at the present time, we can determine with the same accuracy the position of the Sun in relation to MC of PS at that time and its future evolution. The purpose of this knowledge is that we can very accurately determine abnormal acceleration momenta, and as a result, we can predict abnormal events in solar activity. As future evolutions of Sun acceleration may be predicted on reasonable terms from astronomical viewpoint, previous trajectories can also be analyzed and past positions of the Sun with important events in the history of solar activity can be correlated, in order to understand what results a certain acceleration in the past compared to the effects observed and recorded by observers can lead to. In this way one can make a calibration of the evolution of solar activity according to its orbital motion.

Unfortunately, at present we do not know how long it takes for the distribution of kinetic energy impulse on Sun's internal structure (how long it takes to change the internal state), that in order to predict what external effects will follow and when they will occur, depending on the size of impulse whose magnitude and time of occurrence are predictable by solar orbital method.

7 Conclusions

1. The dominant couple of AB in our planetary system is Sun-Jupiter, whose MC is outside the Sun at a distance ranging from 82450 km to Jovian aphelion and 10586 km at Jovian perihelion. Following the motion of Jupiter in its elliptical trajectory, the Sun will also run an elliptical motion between these two limits (see Fig. 3.1). We say that this is the contribution of Jupiter to the solar motion in the solar orbital. Likewise, all the planets in PS will each have a contribution to the motion of the Sun whose final position vector is given by the relation 2.8.

2. Maximum limit of solar position vector $r_{MM} = 1.578 \cdot 10^6$ km is the radius of the circular domain in which all the possible positions of the Sun are ranged, this domain being the first attribute of the *solar orbital* abstract object.

3. Sun's motion in its orbital is characterized by several temporal distributions:

- Temporal distribution of x(t) and y(t) coordinates of solar center in relation to the MC of PS;

- Temporal distribution of solar position vector module;
- Temporal distribution of Sun's velocity on its orbital;
- Temporal distribution of solar acceleration;

4. Each temporal distribution may be associated a frequency distribution. We chose as significant the frequency distribution of solar acceleration as the acceleration is on the one hand energy intake (if positive), and on the other hand as impulses with abnormally large values appear, which may affect the internal state of the Sun.

5. Solar acceleration spectral analysis shows the presence of some spectral components with frequencies (periods) derived from the orbital frequencies of the planets in PS, or differences between the Jupiter frequency and other giant planets frequencies. These frequencies are the components of the second attribute of *solar orbital* abstract object;

6. Spectral components of solar orbital are frequencies (periods) of some periodic processes of Sun's motion overall, motion that could influence internal solar processes and therefore can generate known periodicity in solar activity.

7. Solar acceleration abnormal peaks occurring at certain times, can lead to abnormal phenomena of solar activity. If the mathematical modeling of solar motion is consistent with the motion of the planets in real time the momenta of acceleration peak occurrence are predictable, therefore their effects on solar activity also.

8. Currently, according to the data collected by observers since 1755, 23 complete cycles⁹ of sunspots (surnamed solar cycles) have been revealed. The duration of these cycles varies between 9 and 12.6 years, with an average of 10.6 years per cycle. Spectral analysis of Sun's motion in its orbital reveals much more possible cycles, including the one found so far (average of f_3 and f_7 components, first and third component in intensity is 10.3 years). Unfortunately, modern methods and means of solar parameters measurement (especially from satellites) exist only since few decades ago, an interval insufficient for clear prominence of longer cycles, such as the one of 19.9 years for example, or of cyclical variations of other physical measures characteristic for solar activity and which could not be determined in the past.

9. Spatial-temporal modulation of the solar position in relation to MC of PS causes a corresponding modulation of the solar gravitational field as the gravitational interaction is dependent on the distance between AB couples. For giant planets which are far away from the Sun this field variation is not so important, but for those nearby, especially for Mercury, can be one of the causes of the perihelion advance.

⁹ Wikipedia – List of solar cycles

Annex 1 - Distribution of gravitational interactions intensity on AB couples in our planetary system

I able i	^. I		
n	AB Couple	n	AB Couple
1	Sun - Mercury	24	Venus – Pluto
2	Sun – Venus	25	Earth - Mars
3	Sun – Earth	26	Earth - Jupiter
4	Sun - Mars	27	Earth - Saturn
5	Sun – Jupiter	28	Earth - Uranus
6	Sun – Saturn	29	Earth - Neptune
7	Sun – Uranus	30	Earth - Pluto
8	Sun - Neptune	31	Mars - Jupiter
9	Sun – Pluto	32	Mars - Saturn
10	Mercury - Venus	33	Mars - Uranus
11	Mercury - Earth	34	Mars - Neptune
12	Mercury - Mars	35	Mars - Pluto
13	Mercury - Jupiter	36	Jupiter - Saturn
14	Mercury - Saturn	37	Jupiter - Uranus
15	Mercury - Uranus	38	Jupiter - Neptune
16	Mercury - Neptune	39	Jupiter - Pluto
17	Mercury - Pluto	40	Saturn - Uranus
18	Venus - Earth	41	Saturn - Neptune
19	Venus - Mars	42	Saturn - Pluto
20	Venus - Jupiter	43	Uranus - Neptune
21	Venus - Saturn	44	Uranus - Pluto
22	Venus - Uranus	45	Neptune - Pluto
23	Venus - Neptune		



Fig. X.1 Distribution of gravitational interactions intensity on couples in Table X.1

Annex 2 – Sun motion in case of planetary apsides axes rotation

With regard to the conditions in par. 1 on the calculation of planetary trajectories, in which we determined that elliptical trajectories have collinear apsides axes, in this annex we shall analyze the motion of the planets closest to the actual conditions, i.e. with apsides axes inclined with angles shown in the table of planetary orbitals elements in astronomy books¹⁰.

The rotation of apsides axes is equivalent in viewpoint of relations 1.6 and 1.7 with the rotation of coordinate axes with θ angle, after which, compared to the old x and y coordinates, the new x' and y' coordinates are:

$$x' = x\cos\theta - y\sin\theta \quad y' = x\sin\theta + y\cos\theta \tag{X.2.1}$$

which entered into relations 2.9 and 2.10, the new relations of position calculation (Pluto excluded as its contribution is minor, similar to that of Mars) become:

$$x(t) = \sum_{i=1}^{8} \left\lfloor \frac{a_i}{q_i} \left\lfloor \left(\cos\left(\frac{2\pi}{T_i}t\right) - \varepsilon_i \right) \cos\left(\theta_i\right) - \sin\left(\frac{2\pi}{T_i}t\right) \cos\left(arc\sin(\varepsilon_i)\right) \sin(\theta_i) \right\rfloor \right\rfloor \cdot 1.4959787 \cdot 10^8$$
$$y(t) = \sum_{i=1}^{8} \left\lfloor \frac{a_i}{q_i} \left\lfloor \left(\cos\left(\frac{2\pi}{T_i}t\right) - \varepsilon_i \right) \sin(\theta_i) + \sin\left(\frac{2\pi}{T_i}t\right) \cos\left(arc\sin(\varepsilon_i)\right) \cos(\theta_i) \right\rfloor \right\rfloor \cdot 1.4959787 \cdot 10^8$$

whose graphical representation, also for 131072 days (to be compatible with the FFT analysis) is given by Fig. X.2.1 for position, X.2.2 for position vector module, X.2.3 for velocity and X.2.4 for acceleration.



Fig. X.2.1 - Solar trajectory due to all the planets (excluding Pluto)

 ¹⁰ Imke de Pater, Jack J. Lissauer – Planetary Science, Cambridge University Press, 2001 (p. 6)
 *** - Efemeride astronomice pentru anul 2013, Bumbeşti Jiu, 2012 (p. 150)



Fig. X.2.5 –Solar acceleration spectrum for planetary apsides axes rotation

k	Frequency	Freq.*10 ¹²	Amplit.	T	Т	Comments
	[Hz]	[Hz]	comp.	[days]	[years]	
			rel.val.			
1	1.5894*10 ⁻⁹	1589.4	50.8 (51)	7282	19.9	First major component f_{Ju} - f_{Sa} = 1595.7
2	2.2959*10 ⁻⁹	2295.9	30.4 (31.8)	5041.2	13.8	f_{Ju} - f_{U} =2294.2
3	2.4725*10 ⁻⁹	2472.5	66.2 (68.4)	4681.1	12.8	The most important component f_{Ju} - f_{Ne} = 2479.1
4	2.6491*10 ⁻⁹	2649.1	15.5 (18.7)	4369.1	12	Very close to f _{Ju}
5	3.1789*10 ⁻⁹	3178.9	37.6 (28.3)	3640.9	10	
6	3.8853*10 ⁻⁹	3885.3	19.7 (22.7)	2978.9	8.2	
7	4.0619*10 ⁻⁹	4061.9	44.8 (44.8)	2849.4	7.8	
8	$2.9050*10^{-8}$	29050	15.6 (15)	398.4		Contribution of Earth+Moon
9	4.8832*10 ⁻⁸	48832	29 (30)	237		Contribution of Venus
10	1.2892*10 ⁻⁷	128920	6.4 (6.5)	89.8		Contribution of Mercury

Table X.2.1 – Major spectral components in case of apsides axes rotation

In Fig. X.2.5, and Table X.2.1 one can see that the variation in direction of apsides axes does not affect in terms of quality the major spectral components and in terms of quantity the differences are quite small. For comparison, in the column showing the component amplitude the value in the case of apsides axes alignment is included in brackets, thereby it can be seen low influence of rotation of these axes. The most important influence is exerted by apsides axes rotation on the temporal distribution of solar acceleration peaks, i.e. of the moments when important and abnormal events of solar activity may occur.