Composition of second-order energy capacitances

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1 Rules for the composition of second-order energy capacitances

If the mass is a second-order energy capacitance for kinetic energy (see cap. 7), and the capacitance of an electric capacitor or the inductance of an inductor are still second-order energy capacitances (for electrostatic and magnetic energy), it means that the mass and the two EM capacitances are equivalent abstract objects (obviously in terms of energy). But for capacitors and inductors there are clear composition rules of capacitances for elements mounted in series or in parallel. Why wouldn't such rules exist for masses?

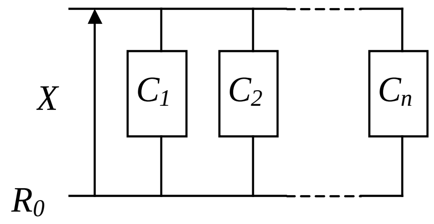

Fig. 1.1

For capacitors mounted in parallel (see figure 1.1), the energy status attribute X (the U voltage evaluated against the absolute reference R_0) is the same for all elements, i.e. the energy status attribute is evenly distributed across the set of connected elements. In this case the total capacitance is the sum of the capacitances of the elements:

$$C_T = C_1 + C_2 + ... + C_n \tag{1.1}$$

For inductors connected in series, the energy status attribute (I current) is again evenly distributed over all the elements of the assembly, in which case the total energy magnetic capacitance (inductance) is also the sum of the individual capacitances:

$$L_T = L_1 + L_2 + ... + L_n \tag{1.2}$$

If we replace the status attribute X in fig. 1.1 by I, the case of series-connected inductors viewed as a distribution of the status attribute has also fig. 1.1 as an equivalent scheme.

Comment 1.1: Fig. 1.1 should not be interpreted as an electric scheme, but as a distribution of the attributes *U, I* or *v*, on *n* material objects with electrostatic, magnetic or kinetic energy capacitance, objects forming a composite object.

In the case of masses and kinetic energy, the situation of the uniformly distributed energy status attribute is the one in which the components of the system in motion, even if they have different masses, move at the same single velocity v, the common global motion speed, with no other specific (individual) components of velocity on each element. In this case, the total energy capacitance (global mass) is the sum of the component masses.

Logically, after these first findings we can say:

Conclusion 1: *If the energy status attribute is evenly distributed over the elements of a composite material system, the resulting second-order energy capacitance of the system is the sum of the capacitances of the elements.*

For capacitors connected in series or inductors connected in parallel, the energy status attribute (capacitor voltage or inductor current) is unevenly distributed. In these cases, the total capacitance is given by the well-known electrotechnical equations:

$$\frac{1}{C_T}=\frac{1}{C_1}+\frac{1}{C_2}+...+\frac{1}{C_n} \tag{1.3}$$

for electrostatic capacitance, respectively:

$$\frac{1}{L_T}=\frac{1}{L_1}+\frac{1}{L_2}+...+\frac{1}{L_n} \tag{1.4}$$

for magnetic capacitance.

Comment 1.2: When discussing the composition of the energy capacitances of material systems (MS), a very important specification must be made - it is the energy capacitances of objects forming a composite object, each component contributing to the capacitance of the composite object either with its entire capacity or with only a part of it depending on how the composite object elements are composed (such as serial/parallel connection in the above examples). As we saw in chap. 3, a composite object has an invariant structure relative to an internal reference system (RS), against which internal attributes are evaluated, this internal RS representing the composite object in its external relations. As a result, the capacitance of the composite object is an external attribute (evaluated against a RS external to the composite object), but assigned to the internal RS.

From the equations 1.3 and 1.4 it can be noted that the total capacitance of the assembly is in both cases less than the sum of the individual capacitances, therefore:

Conclusion 2: *If the elements of the composite object are connected so that the energy status attribute is unevenly distributed, the resulting capacitance of the composite object is always less than the sum of the capacitances of the elements*.

If the reasoning above applies to the masses, we could say that if the elements of an MS move at different velocities, the total mass should be less than the sum of the individual masses.

Note that for capacitors connected in series and inductors connected in parallel there is, however, an evenly distributed attribute, namely the product of the energy capacitance and temporal derivative of the energy status attribute (*I* current for capacitors and *U* voltage for inductors, see equations 1.5 and 1.6). In the case of masses and kinetic energy, if the energy state attribute (*v* velocity) is unevenly distributed, it results that the evenly distributed attribute in this case results is force (see equations 1.7).

For the particular cases of electrostatic (equations 1.5), magnetic (equations 1.6) and kinetic (equations 1.7) energy we have:

$$\Delta Q = I\cdot\Delta t = C\cdot\Delta U \qquad I=\frac{\Delta Q}{\Delta t}=C\cdot\frac{\Delta U}{\Delta t} \qquad K_e^{''}=C \qquad X=U \tag{1.5}$$

$$\Delta B = U\cdot\Delta t = L\cdot\Delta I \qquad U=\frac{\Delta B}{\Delta t}=L\cdot\frac{\Delta I}{\Delta t} \qquad K_e^{''}=L \qquad X=I \tag{1.6}$$

$$\Delta p = F\cdot\Delta t = m\cdot\Delta v \qquad F=\frac{\Delta p}{\Delta t}=m\cdot\frac{\Delta v}{\Delta t} \qquad K_e^{''}=m \qquad X=v \tag{1.7}$$

In a general case, the second-order energy capacitance being $K_e^{''}$, if the energy status attribute is *X*, the specific energy corresponding to this attribute[1] stored in the volume of the material system is:

$$W_x=\int_0^X K_e^{''}XdX=\tfrac{1}{2}K_e^{''}X^2 \tag{1.8}$$

[1] It is strictly the energy the state attribute of which is *X*; other types of energy may also be stored in the same MS, but whose status attributes are different from *X* (such as, for example, rest energy, heat energy, etc.).

2 The MS case consisting of two elements with different masses and velocities

If we have a MS consisting of two bodies orbiting together, maintained by a centripetal attraction force (see fig. 2.1), with individual velocities $\overline{v}_1$ and $\overline{v}_2$, and their common mass centre MC (the internal reference T of the system) has the velocity $\overline{v}_c$, it follows that on the two objects the absolute velocity is unevenly distributed ($\overline{v}_c$ is evenly distributed, but there are specific components $\overline{v}_1$ and $\overline{v}_2$). If the masses of the two bodies are m_1 and m_2 and the distance between them is *d*, the result is two revolution radii $r_1 + r_2 = d$. Compared to an external (absolute) RS, the position vector of MC is given by the equation:

$$\overline{r}_{MC} = \frac{m_1\overline{r}_1 + m_2\overline{r}_2}{m_1 + m_2} \tag{2.1}$$

If the internal reference is set in the MC, between $\overline{r}_1$ and $\overline{r}_2$ we will have the equations:

$$\frac{m_1\overline{r}_1 + m_2\overline{r}_2}{m_1 + m_2} = 0 \text{, i.e. (in the module) } \frac{m_1}{m_1 + m_2} r_1 = \frac{m_2}{m_1 + m_2} r_2 \tag{2.2}$$

$$r_1 = \frac{m_2}{m_1} r_2 = \frac{m_2}{m_1}(d - r_1)\,;\, r_1 + \frac{m_2}{m_1} r_1 = \frac{m_2}{m_1} d\,;\, r_1(1 + \frac{m_2}{m_1}) = \frac{m_2}{m_1} d\,;\, r_1 = \frac{m_2}{m_1 + m_2} d \tag{2.3}$$

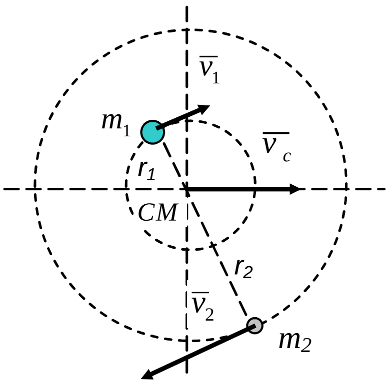

Fig. 2.1

In this case, the invariant internal attributes of the composite object are the *d* distance divided into the two revolution radii r_1 and r_2 relative to MC and angular velocity $\overline{\omega}$ (an attribute evenly distributed over the two elements) that determines the two specific velocities:

$$\overline{v}_1 = \overline{\omega} \times \overline{r}_1 = v_{1x}\overline{i} + v_{1y}\overline{j} = \omega r_1 \cos(\omega t)\overline{i} + \omega r_1 \sin(\omega t)\overline{j} \tag{2.4}$$

and:

$$\overline{v}_2 = \overline{\omega} \times \overline{r}_2 = v_{2x}\overline{i} + v_{2y}\overline{j} = \omega r_2 \cos(\omega t)\overline{i} + \omega r_2 \sin(\omega t)\overline{j} \tag{2.5}$$

where $\overline{i}$, $\overline{j}$ are the unit vectors of X and Y axes.

Attention! Equations 2.4 and 2.5 are valid only if $\overline{v}_c$ is null (the composite object is at rest in relation to the external reference). Because $v = \omega r$, the kinetic energies of the two bodies are:

$$W_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 \omega^2 r_1^2 \tag{2.6}$$

$$W_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_2 \omega^2 r_2^2 \tag{2.7}$$

The centripetal/centrifugal force module (another attribute evenly distributed over the two elements) is given by the equation:

$$\left|F\right| = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} \qquad (2.8)$$

from which on the basis of equations 2.2 and 2.3:

$$\left|F\right| = m_1\omega^2 r_1 = m_2\omega^2 r_2 = \omega^2 \frac{m_1 m_2}{m_1+m_2} d = \omega^2 \frac{m_2 m_1}{m_1+m_2} d = \omega^2 m_c d \qquad (2.9)$$

is obtained.

Comment 2.1: Both in equations 2.8 and 2.9 we are only interested in the centripetal/centrifugal force module. It is obvious that the two forces have opposite directions to their theoretical point of application - the mass centre MC. Note that the module of force *F* (energy link flow) is proportional to the composite mass $m_c = \frac{m_1 m_2}{m_1+m_2}$ if instead of the revolution radii r_1 and r_2 the distance *d* is taken into account.

If the system in fig. 2.1 is moving at velocity $\overline{v}_c = v_{cx}\overline{i} + v_{cy}\overline{j}$ relative to an external reference considered immobile (absolute reference), then the internal velocities $\overline{v}_1$ and $\overline{v}_2$ will be composed of $\overline{v}_c$ resulting in external speeds:

$$\overline{v}_{1e} = \overline{v}_1 + \overline{v}_c = (v_{1x}+v_{cx})\overline{i} + (v_{1y}+v_{cy})\overline{j} = (\omega r_1\cos(\omega t)+v_{cx})\overline{i} + (\omega r_1\sin(\omega t)+v_{cy})\overline{j} \qquad (2.10)$$

$$\overline{v}_{2e} = \overline{v}_2 + \overline{v}_c = (v_{2x}+v_{cx})\overline{i} + (v_{2y}+v_{cy})\overline{j} = (\omega r_2\cos(\omega t)+v_{cx})\overline{i} + (\omega r_2\sin(\omega t)+v_{cy})\overline{j} \qquad (2.11)$$

or taking into account the equations 2.3:

$$\overline{v}_{1e} = (\frac{\omega m_2 d}{m_1+m_2}\cos(\omega t)+v_{cx})\overline{i} + (\frac{\omega m_2 d}{m_1+m_2}\sin(\omega t)+v_{cy})\overline{j} \qquad (2.12)$$

$$\overline{v}_{2e} = (\frac{\omega m_1 d}{m_1+m_2}\cos(\omega t)+v_{cx})\overline{i} + (\frac{\omega m_1 d}{m_1+m_2}\sin(\omega t)+v_{cy})\overline{j} \qquad (2.13)$$

In these conditions, the kinetic energies of the two bodies are:

$$W_{1e} = \frac{1}{2}m_1 v_{1e}^2\,; W_{2e} = \frac{1}{2}m_2 v_{2e}^2 \qquad (2.14$$

and if:

$$\begin{aligned} v_{1e}^2 &= v_{1x}^2 + 2v_{1x}v_{cx} + v_{cx}^2 + v_{1y}^2 + 2v_{1y}v_{cy} + v_{cy}^2 = \omega^2 r_1^2\cos^2(\omega t) + 2\omega r_1 v_{cx}\cos(\omega t) + v_{cx}^2 + \\ &+\omega^2 r_1^2\sin^2(\omega t) + 2\omega r_1 v_{cy}\sin(\omega t) + v_{cy}^2 = \omega^2 r_1^2 + 2\omega r_1(v_{cx}\cos(\omega t) + v_{cy}\sin(\omega t)) + v_c^2 \end{aligned} \qquad (2.15)$$

$$\begin{aligned} v_{2e}^2 &= v_{2x}^2 + 2v_{2x}v_{cx} + v_{cx}^2 + v_{2y}^2 + 2v_{2y}v_{cy} + v_{cy}^2 = \omega^2 r_2^2\cos^2(\omega t) + 2\omega r_2 v_{cx}\cos(\omega t) + v_{cx}^2 + \\ &+\omega^2 r_2^2\sin^2(\omega t) + 2\omega r_2 v_{cy}\sin(\omega t) + v_{cy}^2 = \omega^2 r_2^2 + 2\omega r_2(v_{cx}\cos(\omega t) + v_{cy}\sin(\omega t)) + v_c^2 \end{aligned} \qquad (2.16)$$

then:

$$\begin{aligned} W_{1e} &= \frac{1}{2}m_1(\omega^2 r_1^2 + 2\omega r_1(v_{cx}\cos(\omega t) + v_{cy}\sin(\omega t)) + v_c^2) = \\ &\frac{1}{2}m_1\omega^2 r_1^2 + m_1\omega r_1(v_{cx}\cos(\omega t) + v_{cy}\sin(\omega t)) + \frac{1}{2}m_1 v_c^2 \end{aligned} \qquad (2.17)$$

$$\begin{aligned} W_{2e} &= \frac{1}{2}m_2(\omega^2 r_2^2 + 2\omega r_2(v_{cx}\cos(\omega t) + v_{cy}\sin(\omega t)) + v_c^2) = \\ &\frac{1}{2}m_2\omega^2 r_2^2 + m_2\omega r_2(v_{cx}\cos(\omega t) + v_{cy}\sin(\omega t)) + \frac{1}{2}m_2 v_c^2 \end{aligned} \qquad (2.18)$$

and if we take into account equations 2.2 and 2.3 and we note $m_c = \frac{m_1 m_2}{m_1+m_2}$:

$$W_{1e}=\frac{1}{2m_1}\omega^2 d^2 m_c^2+\omega d m_c(v_{cx}\cos(\omega t)+v_{cy}\sin(\omega t))+\frac{1}{2}m_1 v_c^2 \quad (2.19)$$

$$W_{2e}=\frac{1}{2m_2}\omega^2 d^2 m_c^2+\omega d m_c(v_{cx}\cos(\omega t)+v_{cy}\sin(\omega t))+\frac{1}{2}m_2 v_c^2 \quad (2.20)$$

If we take into account equations 2.6 and 2.7 that express the rest energies of the system and which we note:

$$W_{1r}=\frac{1}{2}m_1\omega^2 r_1^2=\frac{1}{2}m_1\omega^2\left(\frac{m_2 d}{m_1+m_2}\right)^2=\frac{1}{2m_1}\omega^2 d^2 m_c^2\;;W_{2r}=\frac{1}{2m_2}\omega^2 d^2 m_c^2 \quad (2.21)$$

then equations 2.19 and 2.20 become:

$$W_{1e}=W_{1r}+\omega d m_c(v_{cx}\cos(\omega t)+v_{cy}\sin(\omega t))+\frac{1}{2}m_1 v_c^2 \quad (2.22)$$

$$W_{2e}=W_{2r}+\omega d m_c(v_{cx}\cos(\omega t)+v_{cy}\sin(\omega t))+\frac{1}{2}m_2 v_c^2 \quad (2.23)$$

The total energy of the system in motion relative to the external reference is therefore:

$$\begin{aligned}W_e&=W_{1r}+W_{2r}+2\omega d m_c(v_{cx}\cos(\omega t)+v_{cy}\sin(\omega t))+\frac{1}{2}m_1 v_c^2+\frac{1}{2}m_2 v_c^2=\\ &W_r+W_c+2\omega d m_c(v_{cx}\cos(\omega t)+v_{cy}\sin(\omega t))\end{aligned} \quad (2.24)$$

where:

$$W_r=W_{1r}+W_{2r}=\frac{1}{2m_1}\omega^2 d^2 m_c^2+\frac{1}{2m_2}\omega^2 d^2 m_c^2=\frac{1}{2}\omega^2 d^2 m_c^2\left(\frac{1}{m_1}+\frac{1}{m_2}\right)=\frac{1}{2}m_c\omega^2 d^2 \quad (2.25)$$

is the total rest energy of the system, and $W_c=\frac{1}{2}(m_1+m_2)v_c^2$ is the total kinetic energy.

We note that the total rest energy, as it is natural, does not depend on the overall motion with $\bar{v}_c$, but its calculus equation involves the composite mass m_c, and in the expression of total kinetic energy, velocity $\bar{v}_c$ being evenly distributed, the total mass is the sum of the two masses.

The additional term in the equation 2.24 - $2\omega d m_c(v_{cx}\cos(\omega t)+v_{cy}\sin(\omega t))$ - represents the energy flow recirculated between the two elements of the system (the two centripetal forces equal in module, that maintain m_1 and m_2 in their orbital motion). If we take into account the equation 2.9, we can write:

$$2\omega d m_c(v_{cx}\cos(\omega t)+v_{cy}\sin(\omega t))=\frac{2\cos(\omega t)}{\omega}|F|v_{cx}+\frac{2\sin(\omega t)}{\omega}|F|v_{cy} \quad (2.25)$$

The power provided by an energy flow (EF) to a MS (EF intensity through its real separation surface (RSS)) is:

$$P=\left.\frac{\Delta E}{\Delta t}\right|_{RSS}=\bar{F}_e\,\bar{v}_c=\bar{F}_e\frac{\Delta\bar{r}}{\Delta t}=\frac{W}{\Delta t} \quad (2.26)$$

i.e. the *W* mechanical work performed by force $\bar{F}_e$ in the range Δt, against MS inertia. Here $\bar{v}_c$ is the velocity of motion of the MS (namely, the velocity of internal reference T of the MS) obtained by the energy transfer (transaction) from the external EF if the initial velocity of the actuated MS was null, a velocity representing the energy state change of MS as a result of the EF action.

Attention! In the equation 2.26 $\overline{F}_e$ is the external force that moves the MS with velocity $\overline{v}_c$, while in the equation 2.25 it is the centripetal force $|F|$ which maintains the system (link EF). However, the product *Fv* represents the intensity of an energy flow, and in the case of the system in fig. 2.1 the equation 2.25 shows a modulation of the recirculated energy flow (centripetal force) with the amplitude $\frac{2(v_{cx}\cos(\omega t)+v_{cy}\sin(\omega t))}{\omega}$.

3 Conclusions

1. The total second order energy capacitance K''_{eT} of a MS composed of *n* elements over which the energy status attribute *X* has an even distribution is the sum of the energy capacitances of its elements:

$$K''_{eT} = \sum_{i=1}^{n} K''_{ei} \tag{3.1}$$

2. In the case of an uneven distribution of the energy status attribute *X* <u>but an even distribution of the attribute</u> $Y = K''_e \frac{dX}{dt}$ (the product of the energy capacitance and the temporal derivative of the energy status attribute), the capacitance of the composite MS is:

$$\frac{1}{K''_{eT}} = \sum_{i=1}^{n} \frac{1}{K''_{ei}} \tag{3.2}$$

3. If in the case of capacitors and inductors *n* can have values no matter how high in both 3.1 and 3.2 equations, in the case of masses and the equation 3.2 *n* is limited to 2 because the interaction between MS with a mass is always bilateral (by couple). In this case, the attribute *Y* is the force between the two elements (evenly distributed), and the composite mass of the couple with the masses m_1 and m_2 is $m_c = \frac{m_1 m_2}{m_1 + m_2}$.

4. From dimensional point of view the term in the equation 2.25 has the dimension of an energy angular density $F \cdot \frac{LT^{-1}}{\alpha T^{-1}} = F \cdot \frac{L}{\alpha} = \frac{W}{\alpha}$